



CS 4120 Introduction to Compilers

Ross Tate
Cornell University

Lecture 27: Analysis Framework

Preorders

A set X
& a binary relation \leq
reflexivity: $\forall x \in X, x \leq x$
transitivity: $\forall x, y, z \in X, x \leq y \wedge y \leq z$
 $\Rightarrow x \leq z$

Liveness Preorder

Needed!
v
Needed?

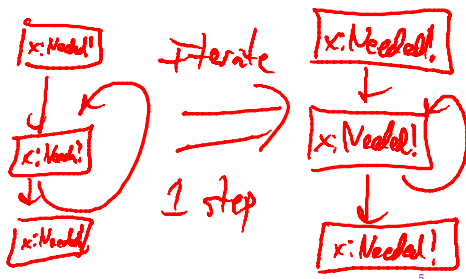
3

Type-Checking Whiles

```
x = "hello"; ← x:Iter(Char)
while (true) ← x:Iter(Char)
  x = x++x; ← x:Iter(Iter(Char))
```

4

Live-Variable Analysis



5

Product Preorder

$\langle X, \leq_x \rangle$ $\langle Y, \leq_y \rangle$

Set is $X \times Y$

$\langle x, y \rangle \in \langle x', y' \rangle$
 $\Leftrightarrow x \leq x' \wedge y \leq y'$

6

Large Products $\langle S, \leq \rangle$ a set N of nodesThe set is $N \rightarrow S$

$$f \leq g \Leftrightarrow \forall n. f(n) \leq g(n)$$

7

Flow Functions $\langle S, \leq \rangle$ A flow function F
is a function $S \rightarrow S$ with
 $\sigma \leq \sigma' \Rightarrow F(\sigma) \leq F(\sigma')$

8

Solution σ such that $F(\sigma) = \sigma$

or more generally

$$F(\sigma) \leq \sigma$$

9

Pointed Preorders $\langle S, \leq, \perp \rangle$

$$\forall \sigma. \perp \leq \sigma$$

10

Iterative AnalysisStart with $\sigma = \perp$

do

$$\sigma := F(\sigma)$$

until $\sigma' \leq \sigma$

$$\sigma := \sigma'$$

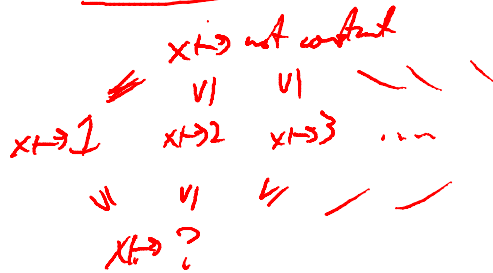
11

Increasing Analysis

$$\perp \leq F\perp \leq F^2\perp \leq F^3\perp \dots$$

12

Non-Finite Lattice



13

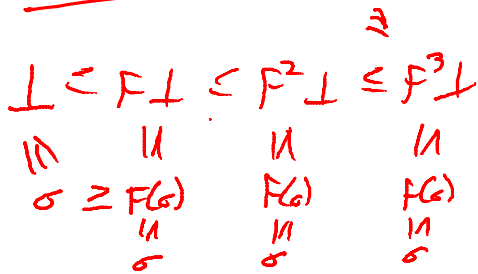
Cowell-Founded

$\forall (\sigma_i)_{i \in \mathbb{N}}$ (increasing)

$\Rightarrow \exists j. \sigma_{j+1} \in \sigma_j$

14

Least Fixpoint



15