We want to translate from a high-level programming into an intermediate representation (IR). This lecture introduces syntax-directed translation as a concise way to formally describe the translation process.

## 1 The target

We are using an IR based on Appel's tree-structured IR:

```
    \(s::=\operatorname{MOVE}\left(e_{\text {dest }}, e_{s r c}\right)\)
        \(\mid \operatorname{EXP}(e)\)
        \(\mid \operatorname{SEQ}\left(s_{1}, \ldots, s_{n}\right)\)
        | JUMP(e)
        \(\mid \operatorname{CJUMP}\left(e, l_{1}, l_{2}\right)\)
        LABEL \((l)\)
        \(e::=\) CONST \((i)\)
        \(\mid \operatorname{TEMP}(t)\)
        | \(O P\left(e_{1}, e_{2}\right)\)
        \(\mid \operatorname{MEM}(e)\)
        \(\mid \mathbf{C A L L}\left(e_{f}, e_{1}, \ldots, e_{n}\right)\)
        | NAME( \(l\) )
        \(\mid \operatorname{ESEQ}(s, e)\)
\(O P::=\) ADD \(\mid\) SUB | MUL | DIV | MOD | SHL | SHR | ASHR
```

We've written these as a grammar, but the grammar also stands for an abstract syntax tree representation of the IR.

## 2 Translation spec

We want to implement a translation from high-level AST nodes to these IR nodes, something like the following methods:

```
class Stmt {
    /** Return an IR statement node implementing this AST node. */
    IRStmtNode translate() {
    }
}
class Expr {
    /** Return an IR expression node implementing this AST node. */
    IRExprNode translate() {
    }
}
```

We will formally describe these two methods with translation functions $\mathcal{S} \llbracket s \rrbracket$ and $\mathcal{E} \llbracket e \rrbracket$. If $s$ is an Iota ${ }_{9}$ statement, $\mathcal{S} \llbracket s \rrbracket$ is its translation into an IR statement node that has the same effect. If $e$ is a Iota ${ }_{9}$ expression,


Figure 1: Memory layouts
$\mathcal{E} \llbracket e \rrbracket$ is its translation into an IR expression node that has the same side effects and evaluates to the same value (or more precisely, the correct IR-level representation of the same value). We will define these two translation functions recursively.

When developing translations, the key is to clearly define the specification of the translation, and to choose the right specification. Once the specification is right, the translation pretty much writes itself. Think of the specification as a contract that must be guaranteed by the actual translation chosen, but can rely on the contract being satisfied by any recursive uses of the same specification.

## 3 Memory layout

The IR is considerably lower-level than the source level: it eliminates all higher-level data structures and control structures, making memory accesses and jumps explicit. This means we have to make some decisions about how built-in types are represented in memory, as depicted in Figure 1.

Arrays need to store their own length, to be able to implement the length operation. We'll represent a value of array type as a reference to element 0 , and store the length just before that in memory.

Functions will be implemented using the C calling conventions for IA-32 (Intel 32-bit) processors: the arguments can be accessed relative to the frame pointer register, which we will write as TEMP $(F P)$ or just $F P$ in the IR. The first argument is at address $F P+8$, the second at $F P+12$, and so on.

To keep things simple for now, we ignore expressions with tuple type. Because the result of these expressions is larger than a single, word-sized IR value, we can't translate them directly to IR expressions.

## 4 Translating expressions

We define the translation of expression $e$ by considering each of the possible syntactic forms that $e$ can take. For each form, there is a single rule that can be chosen to translate the expression. Therefore translation does not involve any searching; it is syntax-directed, just as the type systems we wrote earlier were.

$$
\begin{aligned}
\mathcal{E} \llbracket n \rrbracket & =\operatorname{CONST}(n) \\
\mathcal{E} \llbracket x \rrbracket & =\operatorname{TEMP}(x) \\
\mathcal{E} \llbracket x \rrbracket & =\operatorname{MEM}(\mathbf{N A M E}(x)) \\
\mathcal{E} \llbracket x \rrbracket & =\operatorname{MEM}(\operatorname{ADD}(\operatorname{TEMP}(F P), \operatorname{CONST}(4 * i+4)))
\end{aligned}
$$

(where x is a local variable name)
(where $x$ is a global variable name) (where x is the $i$ th function parameter)

Different kinds of variables are translated differently. Here we've chosen for function parameters to store them in the stack location where arguments are pushed as part of the function calling conventions.

Alternatively we could use a TEMP, and start each function with a prologue that copies all the stack locations into TEMP nodes.

Arithmetic is straightforward to translate. Notice that we have to use the translation function $\mathcal{E} \llbracket e \rrbracket$ recursively on the subexpressions. ${ }^{1}$

$$
\begin{aligned}
\mathcal{E} \llbracket e_{1}+e_{2} \rrbracket & =\mathbf{A D D}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right) \\
\mathcal{E} \llbracket e_{1}-e_{2} \rrbracket & =\mathbf{S U B}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right) \\
\mathcal{E} \llbracket e_{1} * e_{2} \rrbracket & =\mathbf{M U L}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right) \\
\mathcal{E} \llbracket e_{1} / e_{2} \rrbracket & =\mathbf{D I V}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right) \\
\mathcal{E} \llbracket e_{1} \% e_{2} \rrbracket & =\mathbf{M O D}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right)
\end{aligned}
$$

We'll assume that we have the types of expressions available to disambiguate array accesses from function applications. Given this, we can translate array indexing and function calls:

$$
\begin{aligned}
\mathcal{E} \llbracket e_{1} e_{2} \rrbracket=\operatorname{MEM}\left(\mathbf{A D D}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathbf{L S H I F T}\left(\mathcal{E} \llbracket e_{2} \rrbracket, \mathbf{C O N S T}(2)\right)\right)\right) & \left.\left(\text { where } e_{1}: t \llbracket\right]\right) \\
\mathcal{E} \llbracket f\left(e_{1}, \ldots, e_{n}\right) \rrbracket=\mathbf{C A L L}\left(\mathbf{N A M E}(f), \mathcal{E} \llbracket e_{1} \rrbracket, \ldots, \mathcal{E} \llbracket e_{n} \rrbracket\right) & \left(\text { where } f: t->t^{\prime}\right)
\end{aligned}
$$

For example, consider what happens when we translate the source expression $\operatorname{gcd}(x, y-x)$. Using the rules we have so far, we get:

$$
\mathcal{E} \llbracket \operatorname{gcd}(x, y-x) \rrbracket=\mathbf{C A L L}(\mathbf{N A M E}(\operatorname{gcd}), \operatorname{TEMP}(\mathrm{x}), \operatorname{SUB}(\operatorname{TEMP}(\mathrm{y}), \operatorname{TEMP}(\mathrm{x})))
$$

## 5 Translating statements

The translation function $\mathcal{S} \llbracket s \rrbracket$ translates a statement $s$ into a an IR statement.

$$
\begin{aligned}
\mathcal{S} \llbracket x=e \rrbracket & =\operatorname{MOVE}(\mathcal{E} \llbracket x \rrbracket, \mathcal{E} \llbracket e \rrbracket) \\
\mathcal{S} \llbracket e_{1} e_{2}=e_{3} \rrbracket & =\operatorname{MOVE}\left(\mathcal{E} \llbracket e_{1} e_{2} \rrbracket, \mathcal{E} \llbracket e_{3} \rrbracket\right)
\end{aligned}
$$

For statements that involve control, we need to generate labels and use JUMP statements. The statement labels should be fresh: not used anywhere else in the translation.

```
    S \llbracketif (e)s\rrbracket= SEQ(CJUMP (\mathcal{E}\llbrackete\rrbracket, lt, , lf ), (Translation with else is similar)
    LABEL}(\mp@subsup{l}{t}{}),\mathcal{S}\llbrackets\rrbracket
    LABEL}(\mp@subsup{l}{f}{})
S \llbracketwhile (e)s\rrbracket= SEQ(\mathbf{LABEL}(\mp@subsup{l}{h}{}),\mathbf{CJUMP}(\mathcal{E}\llbrackete\rrbracket,\mp@subsup{l}{e}{},\mp@subsup{l}{t}{}),
    LABEL}(\mp@subsup{l}{t}{}),\mathcal{S}\llbrackets\rrbracket
    JUMP(NAME( }\mp@subsup{l}{h}{})\mathrm{ ),
    LABEL}(\mp@subsup{l}{e}{})
```

[^0]
## 6 Translating function definitions

We assume that there is a special TEMP for return values for functions, which we will write TEMP $(R V)$ or just $R V$. Suppose that a function is written as $f\left(x_{1}: t_{1}, \ldots, x_{n}: t_{n}\right)=s$. The function body $s$ is translated to an IR statement as:
$\operatorname{SEQ}(\mathbf{L A B E L}(f)$,
$\mathcal{S} \llbracket s \rrbracket$,
$\left.\mathbf{L A B E L}\left(f_{\text {epilogue }}\right)\right)$

The idea is that when the function is ready to return, the code will jump to the label $f_{\text {epilogue }}$, which is the function epilogue. It will do what is necessary with $R V$ to get the result back to the caller. Therefore we translate return as follows:

$$
\begin{gathered}
\mathcal{S} \llbracket \text { return } \rrbracket=\operatorname{JUMP}\left(f_{\text {epilogue }}\right) \\
\mathcal{S} \llbracket \text { return } e \rrbracket=\underset{\operatorname{SEQ}(\operatorname{MOVE}(\operatorname{TEMP}(R V), \mathcal{E} \llbracket e \rrbracket),}{\left.\operatorname{JUMP}\left(f_{\text {epilogue }}\right)\right)}
\end{gathered}
$$

7 Booleans and control flow
We might be tempted to translate the boolean conjunction operation as follows:

$$
\begin{equation*}
\mathcal{E} \llbracket e_{1} \& e_{2} \rrbracket=\mathbf{A N D}\left(\mathcal{E} \llbracket e_{1} \rrbracket, \mathcal{E} \llbracket e_{2} \rrbracket\right) \tag{BAD!}
\end{equation*}
$$

This doesn't work because we need the \& operator to short-circuit if the first argument is false. We can code this up explicitly using control flow, but the result is rather verbose:

$$
\begin{aligned}
& \mathcal{E} \llbracket e_{1} \& e_{2} \rrbracket=\mathbf{E S E Q}(\mathbf{S E Q}(\operatorname{MOVE}(\mathbf{T E M P}(x), 0), \\
& \mathbf{C J U M P}\left(\mathcal{E} \llbracket e_{1} \rrbracket, l_{1}, l_{f}\right), \\
& \mathbf{L A B E L}\left(l_{1}\right), \operatorname{CJUMP}\left(\mathcal{E} \llbracket e_{2} \rrbracket, l_{2}, l_{f}\right), \\
& \mathbf{L A B E L}\left(l_{2}\right), \operatorname{MOVE}(\operatorname{TEMP}(x), 1) \\
&\left.\mathbf{L A B E L}\left(l_{f}\right)\right), \\
&\operatorname{TEMP}(x))
\end{aligned}
$$

Imagine applying this translation to the statement if $\left(e_{1} \& e_{2}\right) s$. We will obtain a big chunk of IR that combines all the IR nodes from this translation with those of if. Instead, here is a much shorter translation of if $\left(e_{1} \& e_{2}\right) s$ :
$\operatorname{CJUMP}\left(\mathcal{E} \llbracket e_{1} \rrbracket, l_{1}, l_{f}\right)$
$\operatorname{LABEL}\left(l_{1}\right), \operatorname{CJUMP}\left(\mathcal{E} \llbracket e_{2} \rrbracket, l_{2}, l_{f}\right)$
$\mathcal{S} \llbracket s \rrbracket$
$\mathrm{LABEL}\left(l_{f}\right)$
How can we get an efficient translation like this one? The key is to observe that the efficient translation never records the value of $e_{1}$ and $e_{2}$. Instead, it turns their results into immediate control flow decisions. The fact that we arrive at label $l_{1}$ means that $e_{1}$ is true. Because $e_{1}$ and $e_{2}$ appear only as children of CJUMP nodes, we will be able to generate more efficient code when we translate IR to assembly, too.

## 8 Translating booleans to control flow

Therefore, we develop a third syntax-directed translation, which we call $\mathcal{C}$ for "control". The specification of our translation is as follows:
$\mathcal{C} \llbracket e, t, f \rrbracket$ is an IR statement that has all the side effects of $e$, and then jumps to label $t$ if $e$ evaluates to false, and to label $f$ if $e$ evaluates to true.

Given this spec, we can easily translate simple boolean expressions:

$$
\begin{aligned}
\mathcal{C} \llbracket \text { true }, t, f \rrbracket & =\mathbf{J U M P}(t) \\
\mathcal{C} \llbracket \text { false }, t, f \rrbracket & =\mathbf{J U M P}(f)
\end{aligned}
$$

$$
\mathcal{C} \llbracket e, t, f \rrbracket=\mathbf{C J U M P}(\mathcal{E} \llbracket e \rrbracket, t, f) \quad \text { (We use this rule if no other rule applies.) }
$$

The payoff comes for boolean operators, e.g.:

$$
\mathcal{C} \llbracket e_{1} \& e_{2}, t, f \rrbracket=\mathbf{S E Q}\left(\mathcal{C} \llbracket e_{1}, l_{1}, f \rrbracket, \mathbf{L A B E L}\left(l_{1}\right), \mathcal{C} \llbracket e_{2}, l_{2}, f \rrbracket\right)
$$

We can then translate if and while more efficiently by using the $\mathcal{C} \llbracket \rrbracket \rrbracket$ translation instead:

$$
\begin{aligned}
& \mathcal{S} \llbracket \text { if }(e) s \rrbracket=\mathbf{S E Q}\left(\mathcal{C} \llbracket e, l_{1}, l_{2} \rrbracket,\right. \\
& \mathbf{L A B E L}\left(l_{1}\right), \mathcal{S} \llbracket s \rrbracket, \\
&\left.\mathbf{L A B E L}\left(l_{2}\right)\right)
\end{aligned}
$$

$\mathcal{S} \llbracket$ while $(e) s \rrbracket=\mathbf{S E Q}\left(\mathbf{L A B E L}\left(l_{h}\right)\right.$
$\mathcal{C} \llbracket e, l_{1}, l_{e} \rrbracket$,
$\operatorname{LABEL}\left(l_{1}\right), \mathcal{S} \llbracket s \rrbracket$,
$\mathbf{J U M P}\left(l_{h}\right)$,
$\left.\operatorname{LABEL}\left(l_{e}\right)\right)$
Now when we translate if $\left(e_{1} \& e_{2}\right) s$, the result is compact and efficient.


[^0]:    ${ }^{1}$ Because it is always used on proper subterms on the right-hand side, the definition of $\mathcal{E} \llbracket e \rrbracket$ is a well-founded inductive definition.

