Variables vs. Registers/Memory

- **Difference between IR and assembly code:**
  - IR (and abstract assembly) manipulate data in local and temporary variables
  - Assembly code manipulates data in memory/registers

- **During code generation, compiler must account for this difference**

- **Compiler backend must allocate variables to memory or registers in the generated assembly code**
Simple Approach

- **Straightforward solution:**
  - Allocate each variable in activation record
  - At each instruction, bring values needed into registers, perform operation, then store result to memory

```
mov 16(%ebp), %eax
mov 20(%ebp), %ebx
add %ebx, %eax
mov %eax, 24(%ebx)
```

- **Problem:** program execution very inefficient
  - moving data back and forth between memory and registers

```
x = y + z
```

```
x = y + z
```
Register Allocation

- **Better approach** = register allocation: keep variable values in registers as long as possible

- **Best case**: keep a variable’s value in a register throughout the lifetime of that variable
  - In that case, we don’t need to ever store it in memory
  - We say that the variable has been allocated in a register
  - Otherwise allocate variable in activation record
  - We say that variable is spilled to memory

- Which variables can we allocate in registers?
  - Depends on the number of registers in the machine
  - Depends on how variables are being used

- **Main Idea**: cannot allocate two variables to the same register if they are both live at some program point
Register Allocation Algorithm

Hence, basic algorithm for register allocation is:

1. Perform live variable analysis (over abstract assembly code!)
2. Inspect live variables at each program point
3. If two variables are ever in same live set, they can’t be allocated to the same register – they \textit{interfere} with each other
4. Conversely, if two variables do not interfere with each other, they can be assigned the same register. We say they have disjoint live ranges.
**Interference Graph**

- **Nodes** = program variables
- **Edges** = connect variables that interfere with each other

Register allocation = graph coloring

```
b = a + 2;
c = b * b;
b = c + 1;
return b * a;
```
Graph Coloring

• Questions:
  – Can we efficiently find a coloring of the graph whenever possible?
  – Can we efficiently find the optimum coloring of the graph?
  – Can we assign registers to avoid move instructions?
  – What do we do when there aren’t enough colors (registers) to color the graph?
Coloring a Graph

- Let $K =$ number of registers (e.g., take $K=3$)
- Try to color graph with $K$ colors
- **Key operation = Simplify:** find some node with at most $K-1$ edges and cut it out of the graph
Coloring a Graph

- Idea: once coloring is found for simplified graph, removed node can be colored using free color
- Algorithm: simplify until graph contain no nodes
- Add nodes back & assign colors
Stack Algorithm

- **Phase 1: Simplification**
  - Repeatedly simplify graph
  - When a variable (i.e., graph node) is removed, push it on a stack

- **Phase 2: Coloring**
  - Unwind stack and reconstruct the graph as follows:
  - Pop variable from the stack
  - Add it back to the graph
  - Color the node for that variable with a color that it doesn’t interfere with
Stack Algorithm

• Example:

• ...how about:
Failure of Heuristic

• If graph cannot be colored, it will reduce to a graph in which every node has at least K neighbors

• May happen even if graph is colorable in K!

• Finding K-coloring is NP-hard problem (requires search)
Spilling

- Once all nodes have K or more neighbors, pick a node and mark it for possible spilling (storage in activation record).
- Remove it from graph, push it on stack
- Try to pick node not used much, not in inner loop
Optimistic Coloring

• Spilled node may be K-colorable
• Try to color it when popping the stack

• If not colorable, actual spill: assign it a location in the activation record
**Accessing Spilled Variables**

- Need to generate additional instructions to get spilled variables out of activation record and back in again

- **Simple approach:** always reserve extra registers handy for shuttling data in and out

- **Better approach:** rewrite code introducing a new temporary, rerun liveness analysis and register allocation
Rewriting Code

• Example: add v1, v2

• Suppose that v2 is selected for spilling and assigned to activation record location [ebp-24]

• Add new variable (say t35) for just this instruction, rewrite:

  \[
  \text{mov} \ -24(\%ebp), \ t35 \\
  \text{add} \ v1, \ t35
  \]

  **Advantage:** t35 has short lifetime and doesn’t interfere with other variables as much as v2 did.

• Now rerun algorithm; fewer or no variables will spill.
Putting Pieces Together

- Simplify
- Potential Spill
- Optimistic coloring
- Actual Spill

Simplification
Coloring
Precolored Nodes

• Some variables are pre-assigned to registers
  - mul instruction has
    use[I] = eax, def[I] = { eax, edx }
  - result of function call returned in eax

• To properly allocate registers, treat these register uses as special temporary variables and enter into interference graph as precolored nodes
Precolored Nodes

• **Simplify.** Never remove a pre-colored node --- it already has a color, i.e., it is a given register

• **Coloring.** Once simplified graph is all colored nodes, add other nodes back in and color them using precolored nodes as starting point
Optimizing Move Instructions

• Code generation produces a lot of extra mov instructions
  
  \text{mov t5, t9}

• If we can assign t5 and t9 to same register, we can get rid of the mov --- effectively, copy elimination at the register allocation level.

• Idea: if t5 and t9 are not connected in inference graph, coalesce them into a single variable; the move will be redundant.
Coalescing

• When coalescing nodes, take union of edges
• Hence, coalescing results in high-degree nodes
• Problem: coalescing nodes can make a graph uncolorable
**Conservative Coalescing**

- Conservative = ensure that coalescing doesn’t make the graph non-colorable (if it was colorable before)
- Coalesce a and b only if resulting node a/b has fewer than K neighbors of significant degree (●)
  - Safe because we can simplify graph by removing neighbors with insignificant degree (○), then remove coalesced node and get the same graph as before
Conservative Coalescing

- Conservative = ensure that coalescing doesn’t make the graph non-colorable (if it was colorable before)
- Coalesce a and b if resulting node a/b has fewer than K neighbors of significant degree (●)
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Conservative Coalescing

- Conservative = ensure that coalescing doesn’t make the graph non-colorable (if it was colorable before)
- Alternative approach: coalesce a and b if for every neighbor t of a: either t already interferes with b; or t has insignificant degree
  - Safe because removing insignificant neighbors with coalescing yields a subgraph of the graph obtained by removing those neighbors without coalescing
Conservative Coalescing

- Conservative = ensure that coalescing doesn’t make the graph non-colorable (if it was colorable before)
- Alternative approach: coalesce a and b if for every neighbor t of a: either t already interferes with b; or t has insignificant degree
  - Safe because removing insignificant neighbors with coalescing yields a subgraph of the graph obtained by removing those neighbors without coalescing
Simplification + Coalescing

• Consider M = set of move-related nodes (which appear in the source or destination of a move instruction) and N = all other variables
• Start by simplifying as many nodes as possible from N
• Coalesce some pairs of move-related nodes using conservative coalescing; delete corresponding mov instruction(s)
• Coalescing gives more opportunities for simplification: coalesced nodes may be simplified
• If can neither simplify nor coalesce, take a node f in M and freeze all the move instructions involving that variable; i.e., change all f-related nodes from M to N; go back to simplify.
• If all nodes frozen, no simplify possible, spill a variable
Full Algorithm

- Simplify
- Coalesce
- Freeze
- Potential Spill
- Optimistic coloring
- Actual Spill

Simplification

Coloring
Overall Code Generation Process

- Start with low-level IR code
- Build DAG of the computation
  - Access global variables using static addresses
  - Access function arguments using frame pointer
  - Assume all local variables and temporaries are in registers (assume unbounded number of registers)
- Generate abstract assembly code
  - Perform tiling of DAG
- Register allocation
  - Live variable analysis over abstract assembly code
  - Assign registers and generate assembly code
Example

Program

array[int] a

function f:(int x) {
    int i;
    ...
    a[x+i] = a[x+i] + 1;
    ...
}

Low IR

t1 = x+i
t1 = t1*4
t1 = $a+t1
t2 = *t1
t2 = t2+1
t3 = x+i
t3 = t3*4
t3 = $a+t3
*t3 = t2
Accesses to Function Arguments

t1 = x+i
t1 = t1*4
t1 = $a+t1
t2 = *t1
t2 = t2+1
t3 = x+i
t3 = t3*4
t3 = $a+t3
*t3 = t2

t4 = ebp+8
t5 = *t4
t1 = t5+i
t1 = t1*4
t1 = $a+t1
t2 = *t1
t2 = t2+1
t6=ebp+8
t7 = *t6
t3 = t7+i
t3 = t3*4
t3 = $a+t3
*t3 = t2
DAG Construction

\[
\begin{align*}
t4 &= ebp+8 \\
t5 &= \ast t4 \\
t1 &= t5+i \\
t1 &= t1\ast 4 \\
t1 &= a+t1 \\
t2 &= \ast t1 \\
t2 &= t2+1 \\
t6 &= ebp+8 \\
t7 &= \ast t6 \\
t3 &= t7+i \\
t3 &= t3\ast 4 \\
t3 &= a+t3 \\
\ast t3 &= t2
\end{align*}
\]
Tiling

- Find tiles
  - Maximal Munch
  - Dynamic programming

- Temporaries to transfer values between tiles

- No temporaries inside any of the tiles
Abstract Assembly Generation

Abstract Assembly

mov $a, r1
mov 8(%ebp), r3
mov i, r2
add r3, r2
add $1, (r1,r2,4)
Register Allocation

Abstract Assembly

`mov $a, r1`
`mov 8(%ebp), r3`
`mov i, r2`
`add r3, r2`
`add $1, (r1,r2,4)`

Live Variables

{ebp, i}
`mov $a, r1`
{ebp,r1,i}
`mov 8(%ebp), r3`
{r1, r3, i}
`mov i, r2`
{r1,r2,r3}
`add r3, r2`
{r1,r2}
`add $1, (r1,r2,4)`
{}`
Register Allocation

Live Variables

{%ebp, i}
mov $a, r1
{%ebp,r1,i}
mov 8(%ebp), r3
{r1, r3, i}
mov i, r2
{r1,r2,r3}
add r3, r2
{r1,r2}
add $1, (r1,r2,4)
{}

• Build interference graph

i — r3
\/
\/
\/
\/
%ebp— r1 — r2

• Allocate registers:
eax: r1, ebx: r3
i, r2 spilled to memory
Register allocation results:

eax: r1; ebx: r3; i, r2 spilled to memory
Where We Are

Source Program

Optimized Assembly Code