CS412/413

Introduction to Compilers
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Lecture 3: Finite Automata
25 Jan 08

Outline

- RE review
- Construction of lexing automaton
  - DFAs, NFAs
  - DFA simulation
  - RE $\Rightarrow$ NFA conversion
  - NFA $\Rightarrow$ DFA conversion
  - (to be continued for set of prioritized REs)

Concepts

- Tokens: values representing lexical units of a program
  - May represent single character strings (“if”, “+”)
  - May represent set of strings (identifier, number)

- Regular expressions (RE): concise descriptions of tokens
  - Each regular expression R describes language L(R), a set of strings corresponding to a given class of tokens

Regular Expressions

- If R and S are regular expressions, so are:

a	for any character a
ε	empty string
∅	the empty set
R|S	(alternation: "R or S")
RS	(concatenation: "R followed by S")
R*	(Kleene closure: "zero or more R's")

Regular Expression Extensions

- If R is a regular expressions, so are:

R?	= ε | R (zero or one R)
R+	= RR* (one or more R's)
(R)	= R (no effect: grouping)
[abc]	= a|b|c (any of the listed)
[a-e]	= a|b|...| e (character ranges)
[^ab]	= c|d|... (anything but the listed chars)
name = R	named abbreviation

Automatic Lexer Generators

- Input: token spec
  - list of regular expressions in priority order
  - associated action for each RE (generates appropriate kind of token, other bookkeeping)

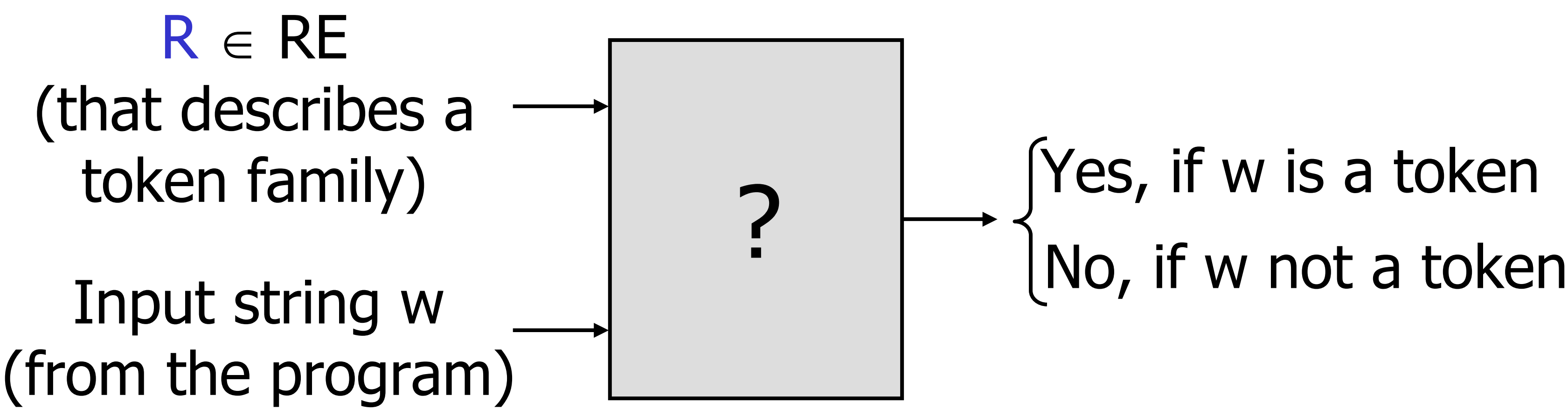
- Output: lexer program
  - program that reads an input stream and breaks it up into tokens according to the REs (or reports lexical error -- "Unexpected character")

Example: JLex

```
%%
digits = 0|[1-9][0-9]*
letter = [A-Za-z]
identifier = {letter}({letter}|[0-9_])*
whitespace = [\ \t\n\r]+
%%
{whitespace}    {/* discard */}
{digits}        { return new Token(INT, Integer.parseInt(yytext()); }
"if"            { return new Token(IF, yytext()); }
"while"         { return new Token(WHILE, yytext()); }
…
{identifier}    { return new Token(ID, yytext()); }
```

How To Use Regular Expressions

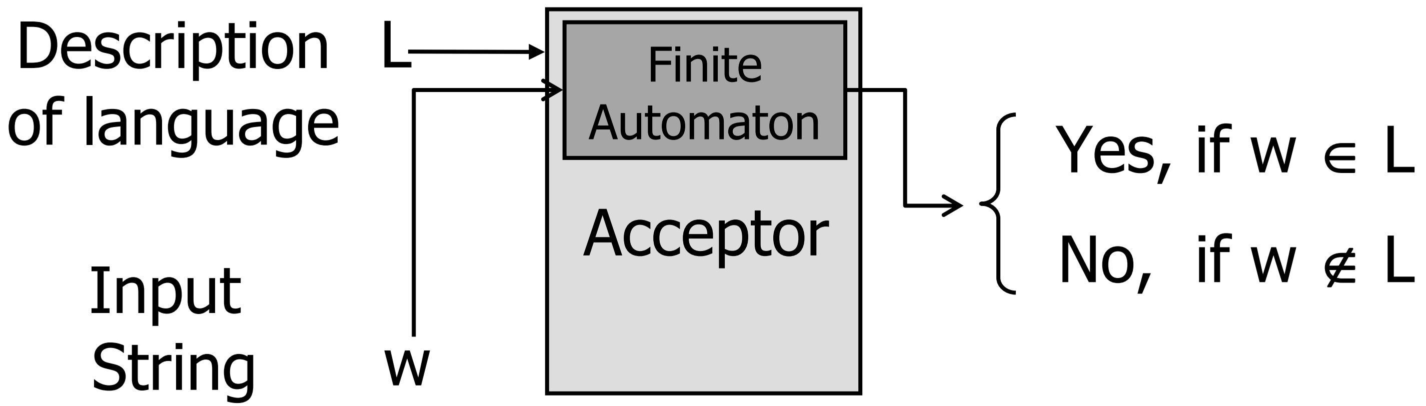
- Given R ∈ RE and input string w, need a mechanism to determine if w ∈ L(R)



- Such a mechanism is called an acceptor

Acceptors

- Acceptor determines if an input string belongs to a language L



- Finite Automata are acceptors for languages described by regular expressions

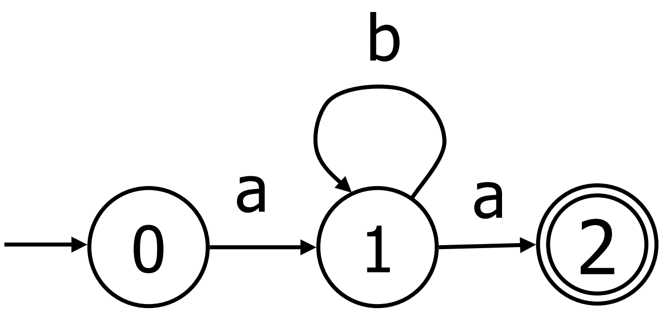
Finite Automata

- Informally, finite automaton consist of:
  - A finite set of states
  - Transitions between states
  - An initial state (start state)
  - A set of final states (accepting states)

- Two kinds of finite automata:
  - Deterministic finite automata (DFA): the transition from each state is uniquely determined by the current input character
  - Non-deterministic finite automata (NFA): there may be multiple possible choices, and some "spontaneous" transitions without input

DFA Example

- Finite automaton that accepts the strings in the language denoted by regular expression ab*a
  - A graph



  - A transition table

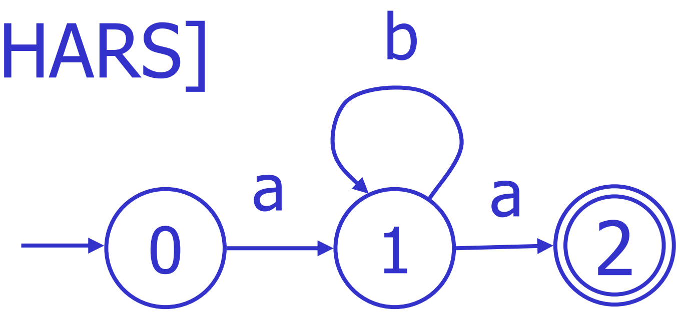
	a	b
0	1	Error
1	2	1
2	Error	Error

Simulating the DFA

- Determine if the DFA accepts an input string

```
trans_table[NSTATES][NCHARS]
accept_states[NSTATES]
state = INITIAL

while (state != Error) {
   c = input.read();
   if (c == EOF) break;
   state = trans_table[state][c];
}
return (state!=Error) && accept_states[state];
```

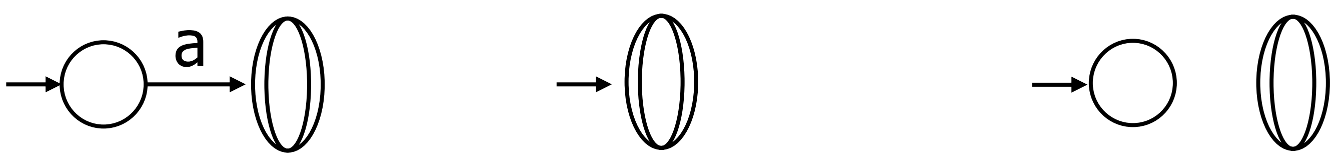


RE ⇒ Finite automaton?

- Can we build a finite automaton for every regular expression?

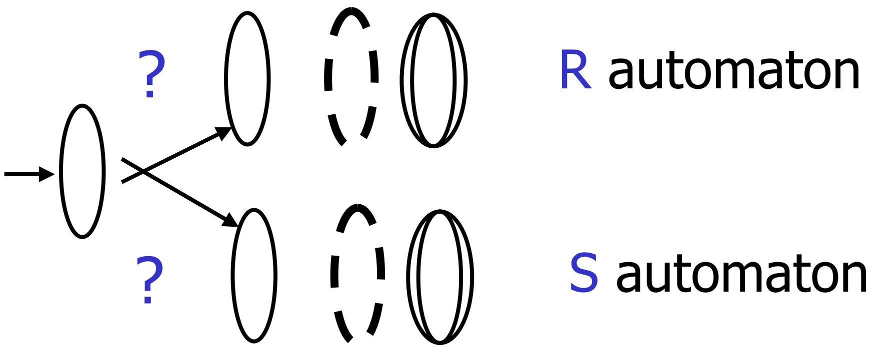
- Strategy: build the finite automaton inductively, based on the definition of regular expressions

a ε ∅

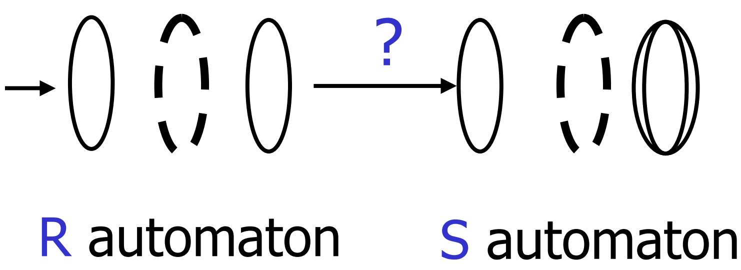


RE ⇒ Finite automaton?

- Alternation R|S



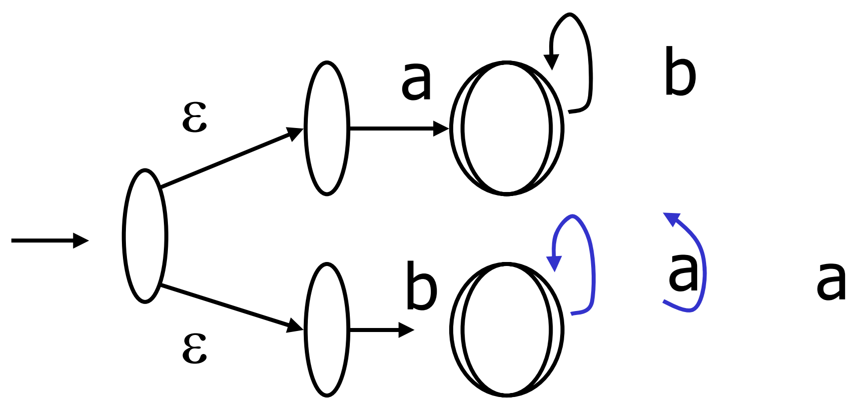
- Concatenation: RS



NFA Definition

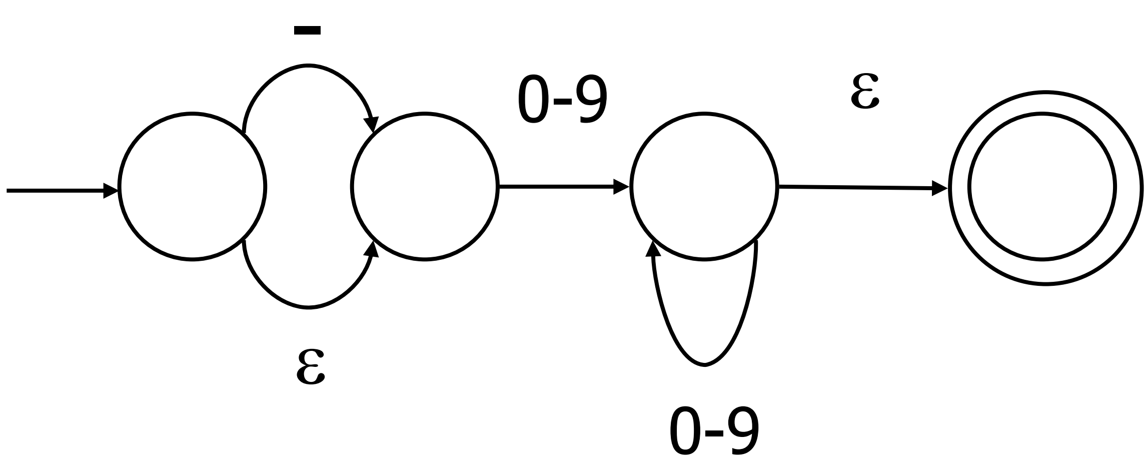
- A non-deterministic finite automaton (NFA) is an automaton where:
  - There may be ε-transitions (transitions that do not consume input characters)
  - There may be multiple transitions from the same state on the same input character

Example:



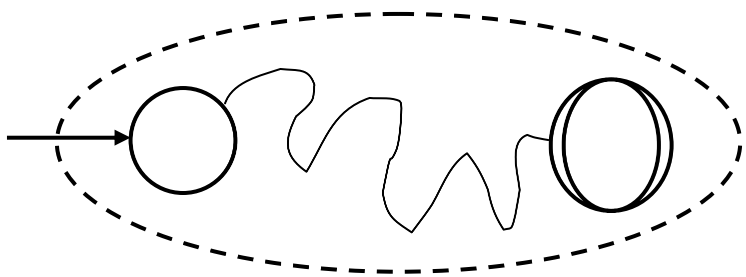
RE ⇒ NFA intuition

-?[0-9]+



NFA construction (Thompson)

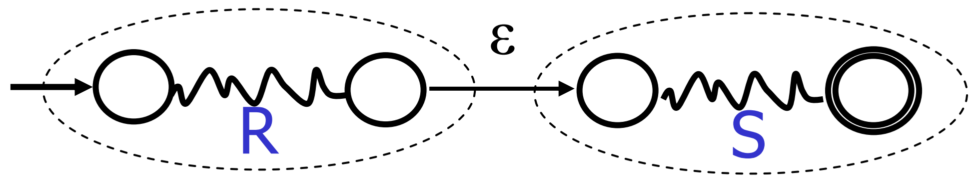
- NFA only needs one stop state (why?)
- Canonical NFA form:



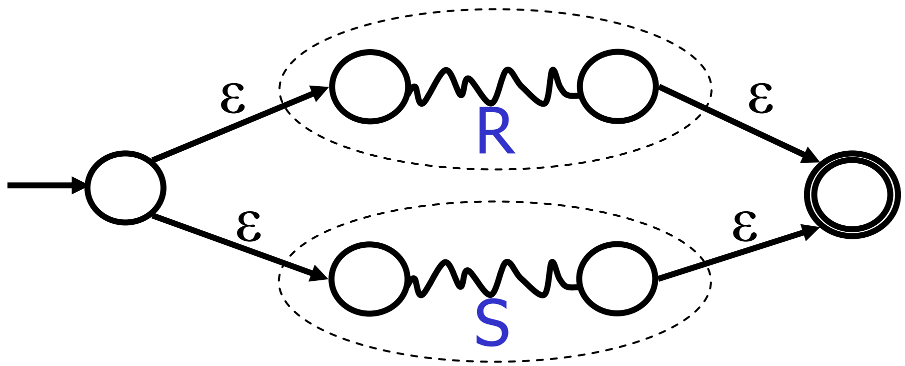
- Use this canonical form to inductively construct NFAs for regular expressions

Inductive NFA Construction

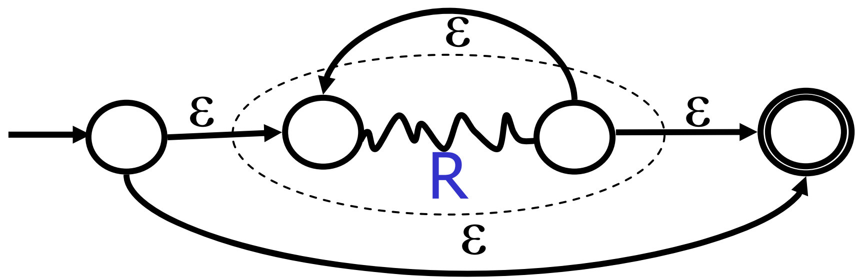
RS



R|S

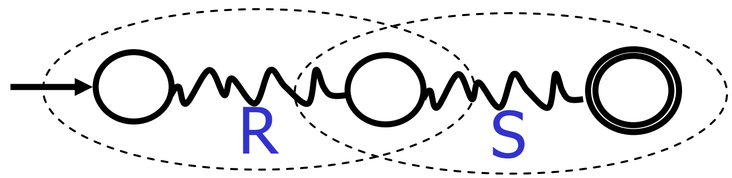


R*

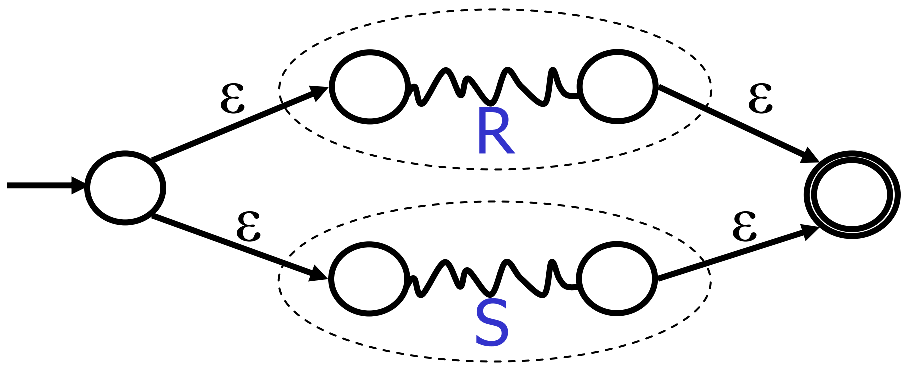


Inductive NFA Construction

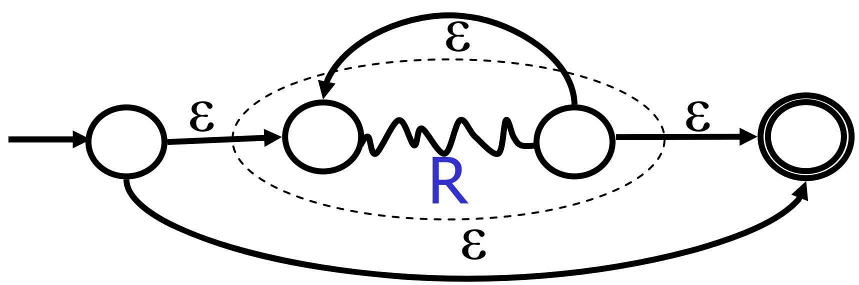
RS



R|S



R*



DFA vs NFA

- DFA: action of automaton on each input symbol is fully determined
  - obvious table-driven implementation
- NFA:
  - automaton may have choice on each step
  - automaton accepts a string if there is any way to make choices to arrive at accepting state / every path from start state to an accept state is a string accepted by automaton
  - not obvious how to implement!

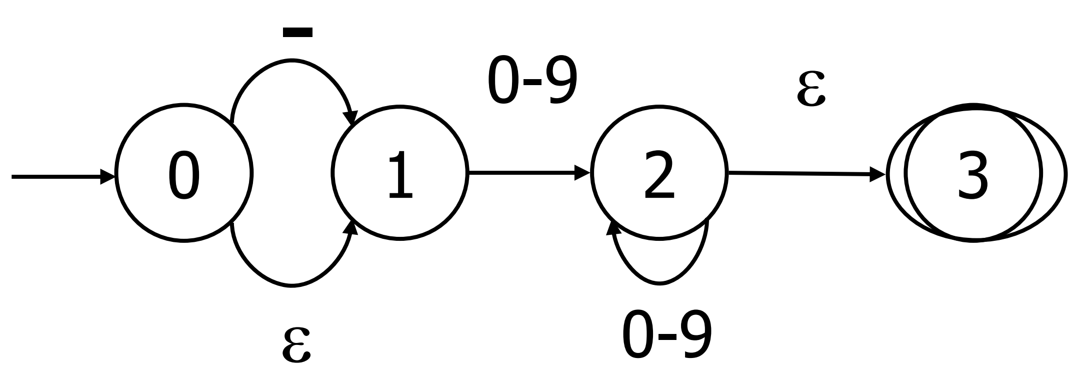
Simulating an NFA

- Problem: how to execute NFA?

  "strings accepted are those for which there is some corresponding path from start state to an accept state"
- Solution: search all paths in graph consistent with the string in parallel
  - Keep track of the subset of NFA states that search could be in after seeing string prefix
  - "Multiple fingers" pointing to graph

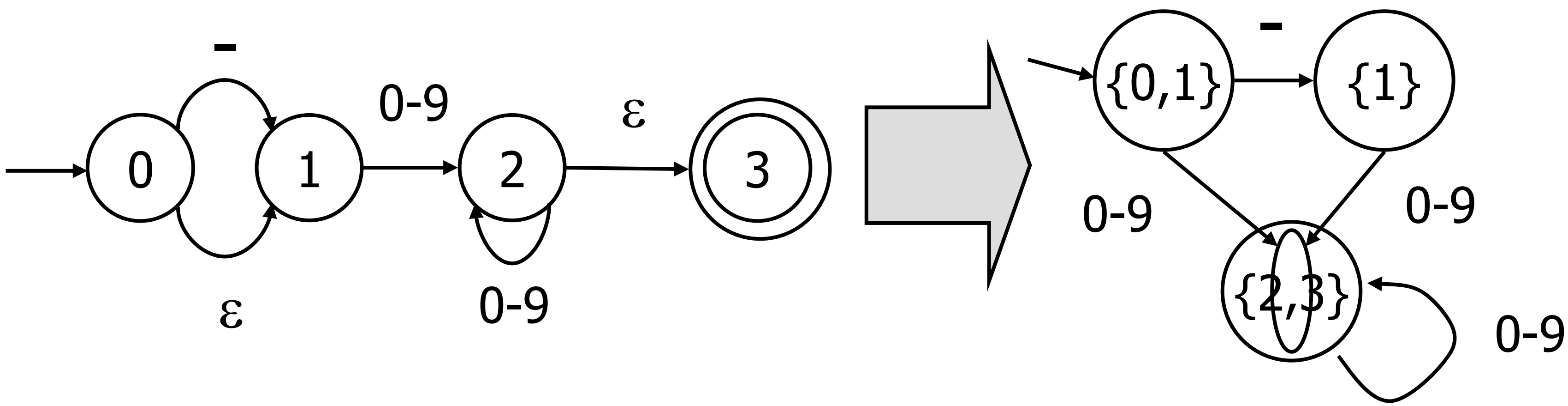
Example

- Input string: -23
- NFA states:
  {0,1}
  {1}
  {2, 3}
  {2, 3}



NFA → DFA conversion

- Can convert NFA directly to DFA by same approach
- Create one DFA state for each distinct subset of NFA states that could arise
- States: {0,1}, {1}, {2, 3}



- Called the "subset construction"

Algorithm

- For a set S of states in the NFA, compute ε-closure(S) = set of states reachable from states in S by one or more ε-transitions

  T = S

  Repeat T = T U {s' | s∈T, (s,s') is ε-transition}

  Until T remains unchanged

  ε-closure(S) = T

- For a set S of ε-closed states in the NFA, compute DFAedge(S,c) = the set of states reachable from states in S by transitions on symbol c and ε-transitions

  DFAedge(S,c) = ε-closure({ s' | s∈S, (s,s') is c-transition})

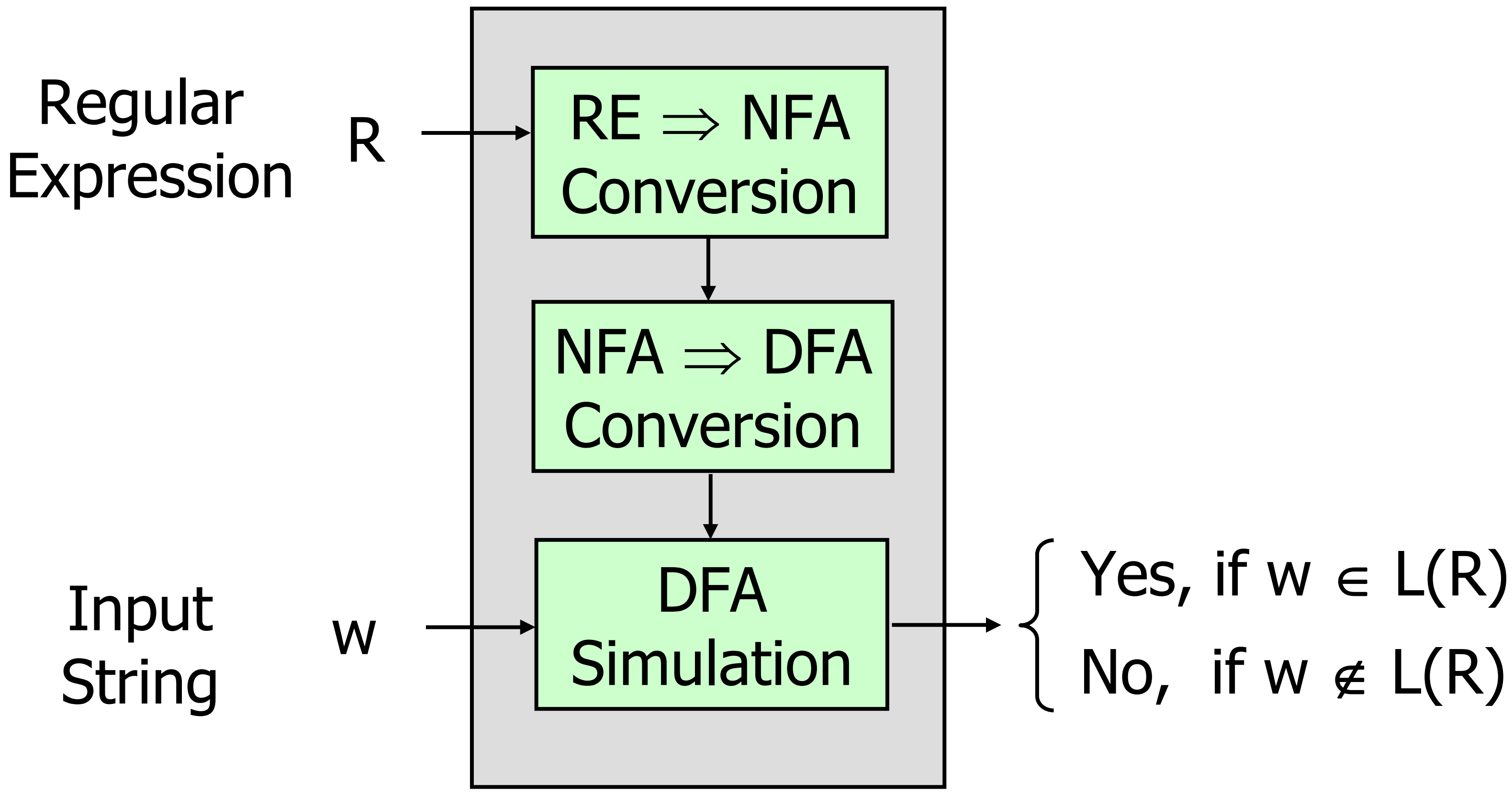
Algorithm

```
DFA-initial-state = ε-closure(NFA-initial-state)
Worklist = { DFA-initial-state }

While ( Worklist not empty )
      Pick state S from Worklist
      For each character c
            S' = DFAedge(S,c)
            if  (S' not in DFA states)
                  Add S' to DFA states and worklist
            Add an edge (S, S') labeled c in DFA

For each DFA-state S
      If S contains an NFA-final state
            Mark S as DFA-final-state
```

Putting the Pieces Together



See Also (on web)

Regular Expression Matching Can Be Simple And Fast (but is slow in Java, Perl, PHP, Python, Ruby, ...), Russ Cox, January 2007