CS412/413

Introduction to Compilers
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Lecture 5: Context-Free Grammars
29 Jan 03

Outline

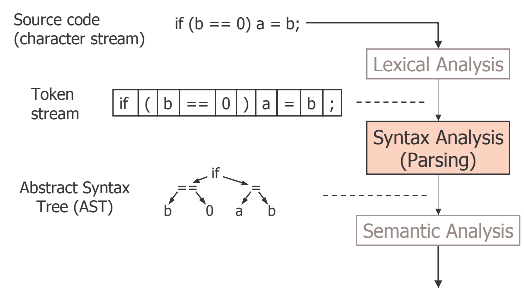
- Context-Free Grammars (CFGs)
- Derivations
- Parse trees and abstract syntax
- Ambiguous grammars

Lexical Analysis

- Translates the program (represented as a stream of characters) into a sequence of tokens
- Uses regular expressions to specify tokens
- Uses finite automata for the translation mechanism

- Lexical analyzers are also referred to as lexers or scanners

Where We Are

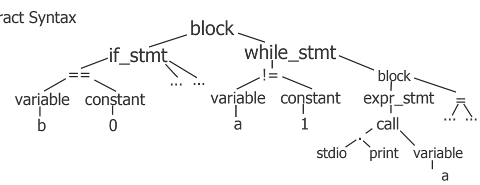


Syntax Analysis Example

Source code
(token stream)

```
{
  if (b == (0)) a = b;
  while (a != 1) {
      stdio.print(a);
      a = a - 1;
  }
}
```

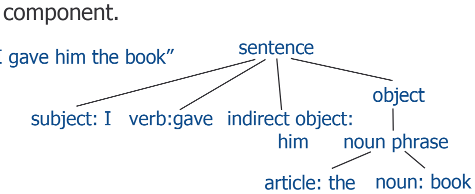
Abstract Syntax
Tree



Parsing Analogy

- Syntax analysis for natural languages:
  recognize whether a sentence is grammatically well-formed & identify the function of each component.

"I gave him the book"



Syntax Analysis Overview

- Goal: determine if the input token stream satisfies the syntax of the program
- What we need for syntax analysis:
  - An expressive way to describe the syntax
  - An acceptor mechanism that determines if the input token stream satisfies that syntax description
- For lexical analysis:
  - Regular expressions describe tokens
  - Finite automata = acceptors for regular expressions

Why Not Regular Expressions?

- Regular expressions can expressively describe tokens
  - easy to implement, efficient (using DFAs)
- Why not use regular expressions (on tokens) to specify programming language syntax?
- Reason: they don't have enough power to express the syntax in programming languages
- Example: nested constructs (blocks, expressions, statements)
  - Language of balanced parentheses

    {{}} {}{} {{}{{}{}}}
    {{ }{ {}}
  - We need unbounded counting!

Context-Free Grammars

- Use Context-Free Grammars (CFG):
  - Terminal symbols = token or ε
  - Non-terminal symbols = syntactic variables
  - Start symbol S = special nonterminal
  - Productions of the form LHS → RHS
    - LHS = a single nonterminal
    - RHS = a string of terminals and non-terminals
    - Specify how non-terminals may be expanded

$S \to a\ S\ a$
$S \to T$
$T \to b\ T\ b$
$T \to \varepsilon$

- Language generated by a grammar = the set of strings of terminals derived from the start symbol by repeatedly applying the productions
  - L(G) denotes the language generated by grammar G

Example

- Grammar for balanced-parenthesis language:

  $S \to \{\ S\ \}\ S$
  $S \to \varepsilon$
- 1 nonterminal: S
- 2 terminals "{" and "}"
- Start symbol: S
- 2 productions:
- If a grammar accepts a string, there is a derivation of that string using the productions:

  $S \to \{S\}S \to \{S\}\ \varepsilon \to \{\{S\}\ S\}\ \varepsilon \to \{\{S\}\ \varepsilon\}\ \varepsilon \to \{\{\varepsilon\}\ \varepsilon\}\ \varepsilon$

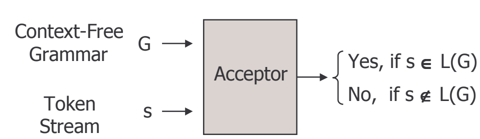
Context-Free Grammars

- Shorthand notation: vertical bar for multiple productions

  $S \to a\ S\ a\ |\ T$
  $T \to b\ T\ b\ |\ \varepsilon$
- Context-free grammars = powerful enough to express the syntax in programming languages
- Derivation = successive application of productions starting from S (the start symbol)
- The acceptor mechanism = determine if there is a derivation for an input token stream

Grammars and Acceptors

- Acceptors for context-free grammars



- Syntax analyzers (parsers) = CFG acceptors which also output the corresponding derivation when the token stream is accepted
  - Various kinds: LL(k), LR(k), SLR, LALR

RE is Subset of CFG

- Inductively build a grammar for each regular expression

ε	$S \rightarrow \varepsilon$
a	$S \rightarrow \mathbf{a}$
$R_1\ R_2$	$S \rightarrow S_1\ S_2$
$R_1 \mid R_2$	$S \rightarrow S_1 \mid S_2$
R_1 *	$S \rightarrow S_1\ S \mid \varepsilon$

where:

G_1 = grammar for R_1, with start symbol S_1
G_2 = grammar for R_2, with start symbol S_2

Sum Grammar

- Grammar:
  $S \rightarrow E + S \mid E$
  $E \rightarrow$ number $\mid$ (S)
- Expanded:
  $S \rightarrow E + S$
  $S \rightarrow E$
  $E \rightarrow$ number
  $E \rightarrow (S)$

  4 productions
  2 non-terminals: S E
  4 terminals: ( ) + number
  start symbol S
- Example accepted input:
  (1 + 2 + (3+4)) + 5

Derivation Example

$S \rightarrow E + S \mid E$
$E \rightarrow$ number | (S)

Derive (1+2+ (3+4))+5:

S → **E** + S → (**S**) + S → (**E** + S)+ S
→ (1 + $\boxed{\textbf{S}}$)+S → (1 + $\boxed{\textbf{E + S}}$)+S
→ (1 + 2 + **S**)+S → (1 + 2 + **E**)+S
→ (1 + 2 + (**S**))+S→ (1 + 2 + (**E** + S))+S
→ (1 + 2 + (3 + **S**))+S
→ (1 + 2 + (3 + **E**))+S
→ (1 + 2+ (3+4))+**S**
→ (1 + 2+ (3+4))+**E**
→ (1 + 2+ (3+4))+5

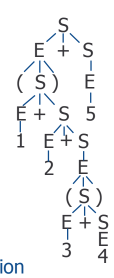
replacement string
non-terminal being expanded

Constructing a Derivation

- Start from S (start symbol)
- Use productions to derive a sequence of tokens from the start symbol
- For arbitrary strings α, β and γ and for a production
  $A \rightarrow \beta$
  a single step of derivation is:
  $\alpha A \gamma \Rightarrow \alpha\beta\gamma$
  (i.e., substitute β for an occurrence of A)
- Example:
  $S \rightarrow E + S$
  $(S + E) + E \rightarrow (E + S + E)+E$

Derivation ⇒ Parse Tree

Parse Tree



- Parse Tree = tree representation of the derivation
- Leaves of tree are terminals
- Internal nodes: non-terminals
- No information about order of derivation steps

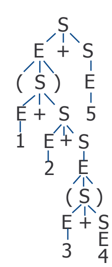
Derivation

S → E + S → (S) + S → (E + S) + S → (1 + S)+S → (1 + E + S) + S → ... → (1 + 2 + (S)) + S→ (1 + 2 + (E + S))+S → ... → (1 + 2 + (3 + E))+S → ... → (1 + 2+ (3+4))+5

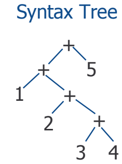
Parse Tree vs. AST

- Parse tree also called “concrete syntax”

Parse Tree (Concrete Syntax)



Abstract Syntax Tree



Discards (abstracts) unneeded information

Derivation order

- Can choose to apply productions in any order; select any non-terminal A: $\alpha A \gamma \Rightarrow \alpha\beta\gamma$
- Two standard orders: left- and right-most -- useful for different kinds of automatic parsing
- Leftmost derivation: In the string, find the left-most non-terminal and apply a production to it

  E + S → 1 + S
- Rightmost derivation: find right-most non-terminal…etc.

  E + S → E + E + S

Example

- S → E + S | E

  E → number | (S)
- Left-most derivation

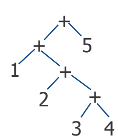
  S →E+S →(S) + S → (E + S)+ S → (1 + S)+S → (1+E+S)+S → (1+2+S)+S → (1+2+E)+S → (1+2+(S))+S → (1+2+(E+S))+S → (1+2+(3+S))+S → (1+2+(3+E))+S → (1+2+(3+4))+S → (1+2+(3+4))+E → (1+2+(3+4))+5
- Right-most derivation

  S →E+S →E+E → E+5 → (S)+5 → (E+S)+5 → (E+E+S)+5 → (E+E+E)+5 → (E+E+(S))+5 → (E+E+(E+S))+5 → (E+E+(E+E))+5 → (E+E+(E+4))+5 → (E+E+(3+4))+5→ (E+2+(3+4))+5 → (1+2+(3+4))+5
- Same parse tree: same productions chosen, diff. order

Parse Trees

- In example grammar, left-most and right-most derivations produced identical parse trees
- + operator associates to right in parse tree regardless of derivation order

(1+2+(3+4))+5



An Ambiguous Grammar

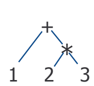
- + associates to right because of right-recursive production S → E + S
- Consider another grammar:

  S → S + S | S * S | number
- Ambiguous grammar = different derivations produce different parse trees

Differing Parse Trees

S → S + S | S * S | number

- Consider expression 1 + 2 * 3
- Derivation 1: **S** → **S** + S → 1 + **S** → 1 + **S** * S → → 1 + 2 * **S** → 1 + 2 * 3
- Derivation 2: **S** → S * **S** → **S** * 3 → S + **S** * 3 → → **S** + 2 * 3 → 1 + 2 * 3



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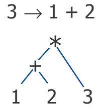
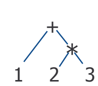


Impact of Ambiguity

- Different parse trees correspond to different evaluations!
- Meaning of program not defined



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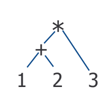

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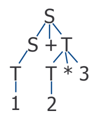
Eliminating Ambiguity

- Often can eliminate ambiguity by adding non-terminals & allowing recursion only on right or left

$S \rightarrow S + T \mid T$
$T \rightarrow T * \mathbf{num} \mid \mathbf{num}$



- T non-terminal enforces precedence
- Left-recursion : left-associativity

Context Free Grammars

- Context-free grammars allow concise syntax specification of programming languages
- A CFG specifies how to convert token stream to parse tree (if unambiguous!)