CS412/413

Introduction to Compilers

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Lecture 31: Subtyping

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Review

- Objects: fields, methods, public/private qualifiers
- Object types: field types + method signatures
  - Interfaces = pure types
  - Objects = types and implementation
- Object inheritance
  - Induces a subtyping relationship S <: T
  - Similar for interfaces
  - Subtyping allows multiple implementations
  - Java: extends, implements
- Type checking
  - Subsumption rule E:T, T<:T' implies E:T'
  - S <: T judgement

Issues

- When are two object/record types identical?
  - Do struct foo { int x,y; } and struct bar { int x,y; } have the same type?
- We know inheritance (i.e. adding methods and fields) induces subtyping relation
- Issues in the presence of subtyping:
  1. Types of records with object fields
     class C1 { Point p; } class C2 { ColoredPoint p; }
  2. Is it safe to allow fields to be written?
  3. Types of functions (methods)
     Point foo(Point p) ColoredPoint bar(ColoredPoint p)

Type Equivalence

- Types derived with constructors have names
- When are record types equivalent?
- When they have the same fields (i.e. same structure)?
  struct point { int x,y; } = struct edge { int n1, n2; } ?
- … or only when they have the same names?
  - Types with the same structure are different if they have different names

Type Equivalence

```
class C1 {
      int x, y;
}
class C2 {
      int x, y;
}
C1 a = new C2();
```

Java: name

```
TYPE t1 = OBJECT
      x,y: INTEGER
END
TYPE t2 = OBJECT
      x,y: INTEGER
END;
VAR a: t1 := NEW(t2);
```

Modula-3: structure

Is this code legal?

Type Equivalence

- Name equivalence: types are equal if they are defined by the same type constructor expression and bound to the same name
  - C/C++ example:
    struct foo { int x; };
    struct bar { int x; }

    struct foo ≠ struct bar
- Structural equivalence: two types are equal if their constructor expressions are equivalent
  - C/C++ example:
    typedef struct foo t1[ ];
    typedef struct foo t2[ ];

    t1 = t2

Declared vs. Implicit Subtyping

Java

```
class C1 {
        int x, y;
}
class C2 extends C1 {
        int z;
}
C1 a = new C2();
```

Modula-3

```
TYPE t1 = OBJECT
        x,y: INTEGER
END
TYPE t2 = OBJECT
        x,y,z: INTEGER
END;
VAR a: t1 := NEW(t2);
```


Named vs. Structural Subtyping

- Name equivalence of types (e.g. Java): direct subtypes explicitly declared; subtype relationships inferred by transitivity
- Structural equivalence of types (e.g., Modula-3): subtypes inferred based on structure of types; extends declaration is optional
- Java: still need to check explicit interface declarations similarly to structural subtyping

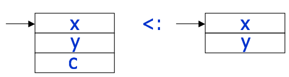
The Subtype Relation

For records:

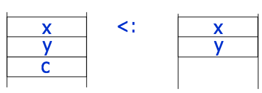
S <: T

{int x; int y; int color; } <: { int x; int y; } ?

- Heap-allocated:



- Stack allocated



Width Subtyping for Records

- Example:

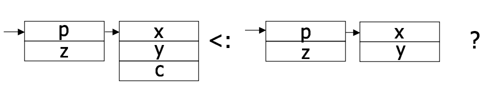
{int x; int y; int color; } ≤ { int x; int y; }

- General rule:

$$\frac{n \le m}{A \vdash \{a_1: T_1, \ldots, a_m: T_m\} <: \{a_1: T_1, \ldots, a_n: T_n\}}$$

Object Fields

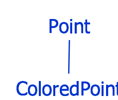
- Assume fields can be objects
- Subtype relations for individual fields
- How does it translate to subtyping for the whole record?
- If ColoredPoint <: Point, allow
  { ColoredPoint p; int z; } <: { Point p; int z; } ?



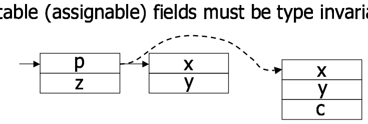
Field Invariance

- Try { p: ColoredPoint; int z; } <: { p: Point; int z; }

```
class C1 { Point p; int z; }
class C2 { ColoredPoint p; int z; }
C1 o1; C2 o2 = new C2();
o1 = o2;
o1.p = new Point( );
o2.p.c = 10;
```

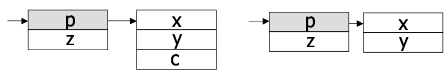


- Mutable (assignable) fields must be type invariant!



Covariance

- Immutable record fields may be type covariant (may allow subtyping)
- Suppose we allow variables to be declared final
  final int x
- Safe:
  { final ColoredPoint p; int z; } <: { final Point p; int z; }



Immutable Record Subtyping

- Rule: corresponding immutable fields may be subtypes; exact match not required

$$\frac{A \vdash T_i <: T_i' \quad (i \in 1..n)}{A \vdash \{a_1: T_1 \ldots a_n: T_n\} <: \{a_1: T_1' \ldots a_n: T_n'\}}$$

$$\frac{n \leq m}{A \vdash \{a_1: T_1, \ldots, a_m: T_m\} <: \{a_1: T_1, \ldots, a_n: T_n\}}$$

Function Subtyping

- Subtyping rules are the same as for records!

  interface List { List rest(int); }
  class SimpleList implements List { SimpleList rest(int); }

- Is this a valid program?
- Is the following subtyping relation correct?
  { rest: int→SimpleList } <: { rest: int→List }

  int→SimpleList <: int→List ?

Signature Conformance

- Subclass method signatures must conform to those of superclass
  - Argument types
  - Return type
  - Exceptions
  - How much conformance is really needed?
- Java rule: arguments and returns must have identical types, may remove exceptions

Function Subtyping

- Mutable fields of a record must be invariant, immutable fields may be covariant
- Object is mostly a record where methods are immutable, non-final fields mutable
- Type of method fields is a function type: $T_1 \times T_2 \times T_3 \rightarrow T_n$
- Subtyping rules for function types will give us subtyping rules for methods

Function Subtyping

```
class Shape {
    int setLLCorner(Point p);
}
class ColoredRectangle extends Shape {
    int setLLCorner(ColoredPoint p);
}
```

- Legal in language Eiffel. Safe?
- Question:

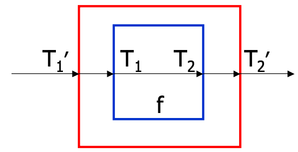
  ColoredPoint → int <: Point → int ?

Function Subtyping

- From definition of subtyping: F: $T_1 \to T_2$ <: F': $T_1' \to T_2'$ if a value of type $T_1 \to T_2$ can be used wherever $T_1' \to T_2'$ is expected
- Requirement 1: whenever result of F' is used, result of F can also be used
  - Implies T_2 <: T_2'
- Requirement 2: any argument to F' must be a valid argument for F
  - Implies T_1' <: T_1

General Rule

- Function subtyping: $T_1 \to T_2$ <: $T_1' \to T_2'$
- Consider function f of type $T_1 \to T_2$:



Contravariance/Covariance

- Function argument types may be contravariant
- Function result types may be covariant

$$\frac{T_1' <: T_1 \qquad T_2 <: T_2'}{T_1 \to T_2 \;<:\; T_1' \to T_2'}$$

- Java is conservative!

{ rest: int→SimpleList } <: { rest: int→List }

Java Arrays

- Java has array type constructor: for any type T, T [ ] is an array of T's
- Java also has subtype rule:

$$\frac{T_1 <: T_2}{T_1[] <: T_2[]}$$

- Is this rule safe?

Java Array Subtype Problems

- Example:

```
Elephant <: Animal
Animal [ ] x;
Elephant [ ] y;
x = y;
x[0] = new Rhinoceros(); // oops!
```

- Covariant modification: unsound
- Java does run-time check!

Unification

- Some rules more problematic: if
- Rule:

$$\frac{A \vdash E : bool \qquad A \vdash S_1 : T \qquad A \vdash S_2 : T}{A \vdash \text{if (E) } S_1 \text{ else } S_2 : T}$$

- Problem: if S_1 has principal type T_1, S_2 has principal type T_2. Old check: $T_1 = T_2$. New check: need principal type T. How to unify T_1 , T_2 ?
- Occurs in Java: ?: operator

General Typing Derivation

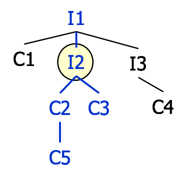
$$\frac{A \vdash E : \text{bool} \qquad \dfrac{A \vdash S_1 : T_1 \quad T_1 <: T}{A \vdash S_1 : T} \qquad \dfrac{A \vdash S_2 : T_2 \quad T_2 <: T}{A \vdash S_2 : T}}{A \vdash \text{if (E) } S_1 \text{ else } S_2 : T}$$

How to pick T ?

Unification

- Idea: unified principal type is least common ancestor in type hierarchy (least upper bound)
- Partial order of types must be a lattice

if (b) new C5() else new C3() : I2



LUB(C3, C5) = I2

Logic: I2 must be same as or a subtype of any type (e.g. I1) that could be the type of both a value of type C3 and a value of type C5

What if no LUB?

Summary

- Type-checking for languages with subtyping
- Subtyping rules often counter-intuitive
  - Types of mutable fields can't be changed (invariant), types of immutable fields can
  - Function return types covariant, argument types contravariant (!)
  - Arrays must be type invariant (like mutable fields)
- Unification requires LUB