CS412/413

Introduction to Compilers
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Lecture 24: Induction Variable Optimizations
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Induction Variables

- An induction variable is a variable in a loop, whose value is a function of the loop iteration number v = f(i)
- In compilers, this a linear function:
  f(i) = c*i + d
- Observation: linear combinations of linear functions are linear functions
  – Consequence: linear combinations of induction variables are induction variables

Induction Variables

- Two categories of induction variables
- Basic induction variables: only incremented in loop body
  i = i + c
  where c is a constant (positive or negative)
- Derived induction variables: expressed as a linear function of an induction variable
  k = c*j + d
  where:
  - either j is basic induction variable
  - or j is derived induction variable in the family of i and:
    1. No definition of j outside the loop reaches definition of k
    2. i is not defined between the definitions of j and k

Families of Induction Variables

- Each basic induction variable defines a family of induction variables
  – Each variable in the family of i is a linear function of i
- A variable k is in the family of basic variable i if:
  1. k = i (the basic variable itself)
  2. k is a linear function of other variables in the family of i:
     k = c*j+d, where j∈Family(i)
- A triple <i, a, b> denotes an induction variable k in the family of i such that: k = i*a + b
  – Triple for basic variable i is <i, 1, 0>

Dataflow Analysis Formulation

- Detection of induction variables: can formulate problem using the dataflow analysis framework
  – Analyze loop sub-graph, except the back edge
  – Analysis is similar to constant folding
- Dataflow information: a function F that assigns a triple to each variable:
  F(k) = <i,a,b>, if k is an induction variable in family of i
  F(k) = ⊥ : k is not an induction variable
  F(k) = ⊤ : don't know if k is an induction variable

Dataflow Analysis Formulation

- Meet operation: if F1 and F2 are two functions, then:

$$(F1 \sqcap F2)(v) = \begin{cases} \langle i,a,b\rangle & \text{if } F1(k)=F2(k)=\langle i,a,b\rangle \\ \bot, & \text{otherwise} \end{cases}$$

  (in other words, use a flat lattice)
- Initialization:
  – Detect all basic induction variables
  – At loop header: F(i) = <i,1,0> for each basic variable i
- Transfer function:
  – consider F is information before instruction I
  – Compute information F' after I

Dataflow Analysis Formulation

- For a definition k = j+c, where k is not basic induction variable
  F'(v) = <i, a, b+c>, if v=k and F(j)=<i,a,b>
  F'(v) = F(v), otherwise
- For a definition k = j*c, where k is not basic induction variable
  F'(v) = <i, a*c, b*c>, if v=k and F(j)=<i,a,b>
  F'(v) = F(v), otherwise
- For any other instruction and any variable k in def[I] :
  F'(v) = ⊥, if F(v) = <k, a, b>
  F'(v) = F(v), otherwise

Strength Reduction

- Basic idea: replace expensive operations (multiplications) with cheaper ones (additions) in definitions of induction variables

```
while (i<10) {
  j = …;   // <i,3,1>
  a[j] = a[j] –2;
  i = i+2;
}
```

⇒

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] –2;
  i = i+2;
  s= s+6;
}
```

- Benefit: cheaper to compute s = s+6 than j = 3*i
  - s = s+6 requires an addition
  - j = 3*i requires a multiplication

General Algorithm

- Algorithm:

For each induction variable j with triple <i,a,b>
whose definition involves multiplication:
1. create a new variable s
2. replace definition of j with j=s
3. immediately after i=i+c, insert s = s+a*c
   (here a*c is constant)
4. insert s = a*i+b into preheader

- Correctness:
  this transformation maintains the invariant that s = a*i+b

Strength Reduction

- Gives opportunities for copy propagation, dead code elimination

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] –2;
  i = i+2;
  s= s+6;
}
```

⇒

```
s = 3*i+1;
while (i<10) {

  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```


Induction Variable Elimination

- Idea: eliminate each basic induction variable whose only uses are in loop test conditions and in their own definitions i = i+c
  - rewrite loop test to eliminate induction variable

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

- When are induction variables used only in loop tests?
  - Usually, after strength reduction
  - Use algorithm from strength reduction even if definitions of induction variables don't involve multiplications

Induction Variable Elimination

- Rewrite test condition using derived induction variables
- Remove definition of basic induction variables (if not used after the loop)

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

⇒

```
s = 3*i+1;
while (s<31) {
  a[s] = a[s] –2;
  s= s+6;
}
```

Induction Variable Elimination

For each basic induction variable i whose only uses are
- The test condition i < u
- The definition of i: i = i + c

For each derived induction variable k in its family, with triple <i,c,d>

Replace test condition i < u with k < c*u+d

Remove definition i = i+c if i is not live on loop exit

Where We Are

- Defined dataflow analysis framework
- Used it for several analyses
  - Live variables
  - Available expressions
  - Reaching definitions
  - Constant folding
- Loop transformations
  - Loop invariant code motion
  - Induction variables
- Next:
  - Pointer alias analysis

Pointer Alias Analysis

- Most languages use variables containing addresses
  - E.g. pointers (C,C++), references (Java), call-by-reference parameters (Pascal, C++, Fortran)
- Pointer aliases: multiple names for the same memory location, which occur when dereferencing variables that hold memory addresses
- Problem:
  - Don't know what variables read and written by accesses via pointer aliases (e.g. *p=y, x=*p, p.f=y, x=p.f, etc.)
  - Need to know accessed variables to compute dataflow information after each instruction

Pointer Alias Analysis

- Worst case scenarios
  - *p = y may write any memory location
  - x = *p may read any memory location
- Such assumptions may affect the precision of other analyses
- Example1: Live variables
  before any instruction x = *p, all the variables may be live
- Example 2: Constant folding
  a = 1; b = 2;*p = 0; c = a+b;
- c = 3 at the end of code only if *p is not an alias for a or b!
- Conclusion: precision of result for all other analyses depends on the amount of alias information available
  - hence, it is a fundamental analysis

Alias Analysis Problem

- Goal: for each variable v that may hold an address, compute the set Ptr(v) of possible targets of v
  - Ptr(v) is a set of variables (or objects)
  - Ptr(v) includes stack- and heap-allocated variables (objects)
- Is a "may" analysis: if $x \in$ Ptr(v), then v may hold the address of x in some execution of the program
- No alias information: for each variable v, Ptr(v) = V, where V is the set of all variables in the program

Simple Alias Analyses

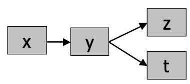
- Address-taken analysis:
  - Consider AT = set of variables whose addresses are taken
  - Then, Ptr(v) = AT, for each pointer variable v
  - Addresses of heap variables are always taken at allocation sites (e.g. x = new int[2], x=malloc(8))
  - Hence AT includes all heap variables
- Type-based alias analysis:
  - If v is a pointer (or reference) to type T, then Ptr(v) is the set of all variables of type T
  - Example: p.f and q.f can be aliases only if p and q are references to objects of the same type
  - Works only for strongly-typed languages

Dataflow Alias Analysis

- Dataflow analysis: for each variable v, compute points-to set Ptr(v) at each program point

- Dataflow information: set Ptr(v) for each variable v
  - Can be represented as a graph $G \subseteq 2^{V \times V}$
  - Nodes = V (program variables)
  - There is an edge v→u if u ∈ Ptr(v)

Ptr(x) = {y}
Ptr(y) = {z,t}



Dataflow Alias Analysis

- Dataflow Lattice: $(2^{V \times V}, \supseteq)$
  - V x V is set of all possible points-to relations
  - "may" analysis: top element is ∅, meet operation is ∪

- Transfer functions: use standard dataflow transfer functions:
  out[I] = (in[I]-kill[I]) U gen[I]

p = addr q	kill[I]={p} x V	gen[I]={(p,q)}
p = q	kill[I]={p} x V	gen[I]={p} x Ptr(q)
p = [q]	kill[I]={p} x V	gen[I]={p} x Ptr(Ptr(q))
[p] = q	kill[I]={}	gen[I]=Ptr(p) x Ptr(q)

For all other instruction, kill[I] = {}, gen[I] = {}

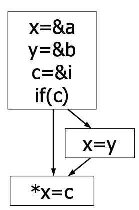
- Transfer functions are monotonic, but not distributive!

Alias Analysis Example

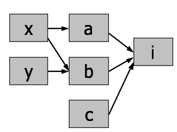
Program

```
x=&a;
y=&b;
c=&i;
if(i) x=y;
*x=c;
```

CFG



Points-to Graph
(at the end of program)



Alias Analysis Uses

- Once alias information is available, use it in other dataflow analyses

- Example: Live variable analysis
  Use alias information to compute use[I] and def[I] for load and store statements:

x = [y]	use[I] = {y}∪Ptr(y)	def[I]={x}
[x] = y	use[I] = {x,y}	def[I]=Ptr(x)