CS412/413

Introduction to Compilers
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Lecture 3: Finite Automata
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Outline

- Regexp review
- DFAs, NFAs
- DFA simulation
- RE-NFA conversion
- NFA-DFA conversion

Regular Expressions

- If R and S are regular expressions, so are:

ε	empty string
a	for any character **a**
R S	(concatenation: "R followed by S")
R | S	(alternation: "R or S")
R *	(Kleene star: "zero or more R's")

Regular Expression Extensions

- If *R* is a regular expressions, so are:

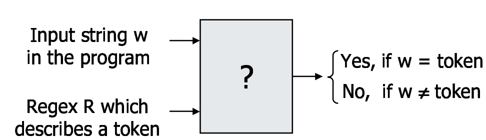
R ?	= ε | R (zero or one R)
R +	= RR* (one or more R's)
(R)	= R (no effect: grouping)
[**abc**]	= a|b|c (any of the listed)
[**a**-**e**]	= a|b|…| e (character ranges)
[^**ab**]	= c|d|…
	(anything but the listed chars)

Concepts

- Tokens = strings of characters representing the lexical units of the programs, such as identifiers, numbers, keywords, operators
  - May represent a unique character string (keywords, operators)
  - May represent multiple strings (identifiers, numers)
- Regular expressions = concise description of tokens
  - A regular expressions describes a set of strings
- Language denoted by a regular expression = the set of strings that it represents
  - L(R) is the language denoted by regular expression R

How To Use Regular Expressions

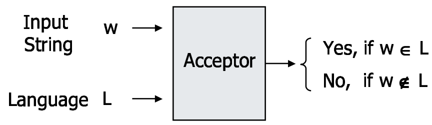
- We need a mechanism to determine if an input string *w* belongs to the language denoted by a regular expression *R*



- Such a mechanism is called an acceptor

Acceptors

- Acceptor = determines if an input string belongs to a language L



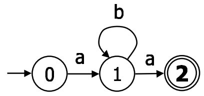
- Finite Automata = acceptor for languages described by regular expressions

Finite Automata

- Informally, finite automata consist of:
  - A *finite* set of states
  - Transitions between states
  - An initial state (start state)
  - A set of final states (accepting state)
- Two kinds of finite automata:
  - Deterministic finite automata (DFA): the transition from each state is uniquely determined by the current input character
  - Non-deterministic finite automata (NFA): there may be multiple possible choices or some transitions do not depend on the input character

DFA Example

- Finite automaton that accepts the strings in the language denoted by the regular expression ab*a
  - A graph



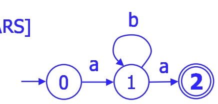
  - A transition table

	a	b
0	1	Error
1	2	1
2	Error	Error

Simulating the DFA

- Determine if the DFA accepts an input string

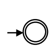
```
trans_table[NSTATES][NCHARS]
accept_states[NSTATES]
state = INITIAL

while (state != ERROR) {
   c = input.read();
   if (c == EOF) break;
   state = trans_table[state][c];
}
return accept_states[state];
```



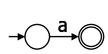
RE → Finite automaton?

- Can we build a finite automaton for every regular expression?
- Strategy: build the finite automaton inductively, based on the definition of regular expressions

ε

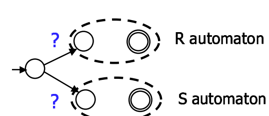


a

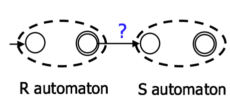


RE → Finite automaton?

- Alternation R | S



- Concatenation: R S

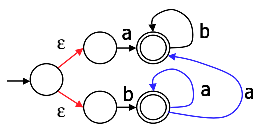


NFA Definition

- A non-deterministic finite automaton (NFA) is an automaton where the state transitions are such that:
  - There may be ε-transitions (transitions which do not consume input characters)
  - There may be multiple transitions from the same state on the same input character

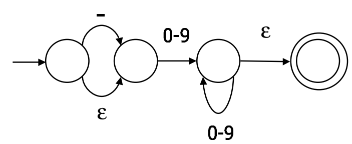
Example:

regexp?



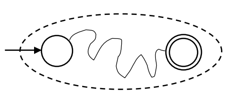
RE ⇒ NFA intuition

-?[0-9]+ (-|ε) [0-9][0-9]*



NFA construction

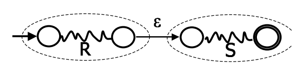
- NFA only needs one stop state (why?)
- Canonical NFA:



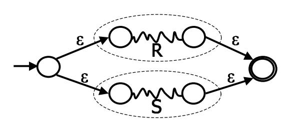
- Use this canonical form to inductively construct NFAs for regular expressions

Inductive NFA Construction

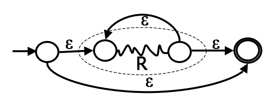
R S



R | S



R*



DFA vs NFA

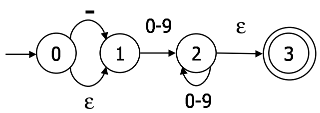
- DFA: action of automaton on each input symbol is fully determined
  - obvious table-driven implementation
- NFA:
  - automaton may have choice on each step
  - automaton accepts a string if there is *any way* to make choices to arrive at accepting state / every path from start state to an accept state is a string accepted by automaton
  - not obvious how to implement!

Simulating an NFA

- Problem: how to execute NFA?
  "strings accepted are those for which there is some corresponding path from start state to an accept state"
- Conclusion: search all paths in graph consistent with the string
- Idea: search paths in parallel
  - Keep track of subset of NFA states that search could be in after seeing string prefix
  - "Multiple fingers" pointing to graph

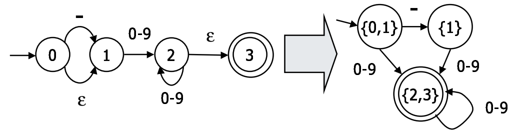
Example

- Input string: -23
- NFA states:
  {0,1}
  {1}
  {2, 3}
  {2, 3}



NFA-DFA conversion

- Can convert NFA directly to DFA by same approach
- Create one DFA for each distinct subset of NFA states that could arise
- States: {0,1}, {1}, **{2, 3}**



Algorithm

- For a set S of states in the NFA, compute ε-closure(S) = the set of states reachable from states in S by ε-transitions

  T = S
  Repeat T = T U {s | s′∈T, (s,s) is ε-transition}
  Until T remains unchanged
  ε-closure(S) = T

- For a set S of states in the NFA, compute DFAedge(S,c) = the set of states reachable from states in S by transitions on character c and ε-transitions

  DFAedge(S,c) = ε-closure({ s | s′∈S, (s′,s) is c-transition})

Algorithm

- Top-level algorithm:

DFA-initial-state = ε-closure(NFA-initial-state)

For each DFA-state S
  For each character c
    S′ = DFAedge(S,c)
    Add an edge (S, S′) labeled with character c in the DFA

For each DFA-state S
  If S contains an NFA-final state
    Mark S as DFA-final-state

Putting the Pieces Together