



# CS 412 Introduction to Compilers

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Lecture 6: AST construction and  
bottom-up parsing  
5 Feb 01

## Administrivia

- Programming Assignment 1 due on Wednesday
- Check class newsgroup `cornell.class.cs412` for answers to frequently asked questions

## Completing the parser

Now we know how to construct a recursive-descent parser for an LL(1) grammar.

Can we use recursive descent to build an abstract syntax tree too?

## Creating the AST

```
abstract class Expr { }
```

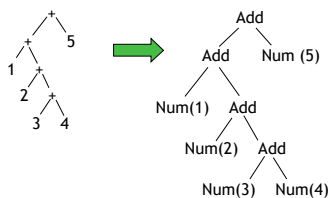
```
class Add extends Expr {
  Expr left, right;
  Add(Expr L, Expr R) { left = L; right = R; }
}
```



```
class Num extends Expr {
  int value;
  Num (int v) { value = v; }
}
```

## AST Representation

$(1 + 2 + (3 + 4)) + 5$



How can we generate this structure during recursive-descent parsing?

## Creating the AST

- Just add code to each parsing routine to create the appropriate nodes!
- Works because parse tree and call tree have same shape
- `parse_S`, `parse_S'`, `parse_E` all return an `Expr`:

```
void parse_E() ⇒ Expr parse_E()
void parse_S() ⇒ Expr parse_S()
void parse_S'() ⇒ Expr parse_S'()
```

## AST creation code

```
Expr parse_E() {
  switch(token) {
  case num: // E → number
    Expr result = Num (token.value);
    token = input.read(); return result;
  case '(': // E → ( S )
    token = input.read();
    Expr result = parse_S();
    if (token != ')') throw new ParseError();
    token = input.read(); return result;
  default: throw new ParseError();
  }
}
```

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## parse\_S

```
Expr parse_S() {
  switch (token) {
  case num:
  case '(':
    Expr left = parse_E();
    Expr right = parse_S();
    if (right == null) return left;
    else return new Add(left, right);
  default: throw new ParseError();
  }
}
```

$$\begin{array}{l} S \rightarrow ES' \\ S' \rightarrow \epsilon \mid +S \\ E \rightarrow \text{num} \mid (S) \end{array}$$

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## Or...an Interpreter!

```
int parse_E() {
  switch(token) {
  case number:
    int result = token.value;
    token = input.read(); return result;
  case '(':
    token = input.read();
    int result = parse_S();
    if (token != ')') throw new ParseError();
    token = input.read(); return result;
    default: throw new ParseError();
  }
}

int parse_S() {
  switch (token) {
  case number:
  case '(':
    int left = parse_E();
    int right = parse_S();
    if (right == 0) return left;
    else return left + right;
  default: throw new ParseError();
  }
}
```

$$\begin{array}{l} S \rightarrow ES' \\ S' \rightarrow \epsilon \mid +S \\ E \rightarrow \text{num} \mid (S) \end{array}$$

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## Grammars

- Have been using grammar for language of “sums with parentheses” e.g.,  $(1+(3+4))+5$

- Simple grammar w/ left associativity:

$$\begin{array}{l} S \rightarrow S + E \mid E \\ E \rightarrow \text{number} \mid (S) \end{array}$$

- LL(1) grammar for same language:

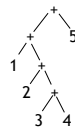
$$\begin{array}{l} S \rightarrow ES' \\ S' \rightarrow \epsilon \mid +S \\ E \rightarrow \text{number} \mid (S) \end{array}$$

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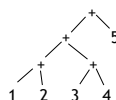
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## Left vs. Right Recursion

Right recursion : right-associative

$$\begin{array}{l} S \rightarrow ES' \\ S' \rightarrow \epsilon \mid +S \end{array} \approx \begin{array}{l} S \rightarrow E + S \\ S \rightarrow E \end{array}$$


Left recursion : left-associative

$$\begin{array}{l} S \rightarrow S + E \\ S \rightarrow E \end{array}$$


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## Left-recursive vs Right-recursive

- Left-recursive grammars don't work with top-down parsing; arbitrary amount of look-ahead needed

derived string    lookahead    read/unread

derived string	lookahead	read/unread
$S$	1	$1 + 2 + 3 + 4$
$S + E$	1	$1 + 2 + 3 + 4$
$S + E + E$	1	$1 + 2 + 3 + 4$
$S + E + E + E$	1	$1 + 2 + 3 + 4$
$E + E + E + E$	1	$1 + 2 + 3 + 4$
$1 + E + E + E$	2	$1 + 2 + 3 + 4$
$1 + 2 + E + E$	3	$1 + 2 + 3 + 4$
$1 + 2 + 3 + E$	4	$1 + 2 + 3 + 4$
$1 + 2 + 3 + 4$	\$	$1 + 2 + 3 + 4$

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## How to create an LL(1) grammar

- Write a right-recursive grammar

$$S \rightarrow E + S$$

$$S \rightarrow E$$

- Left-factor* common prefixes, place suffix in new non-terminal

$$S \rightarrow E S'$$

$$S' \rightarrow \epsilon$$

$$S' \rightarrow + S$$

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## EBNF

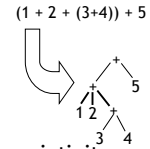
- Extended Backus-Naur Form**: allows some regular expression syntax on RHS

- \*, +, (, ? operators (Iota spec: ? = [ ])
- BNF: | operator at top level

$$S \rightarrow E S'$$

$$S' \rightarrow \epsilon \mid + S$$

$$S \rightarrow E ( + E )^*$$



- EBNF version: no position on + associativity

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## Top-down parsing EBNF

- Recursive-descent code can directly implement the EBNF grammar:

$$S \rightarrow E ( + E )^*$$

```
void parse_S () { // parses sequence of E + E + E ...
    parse_E ();
    while (true) {
        switch (token) {
            case '+': token = input.read(); parse_E();
                    break;
            case ')': case EOF: return;
            default: throw new ParseError();
        }
    }
}
```

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## Reassociating the AST

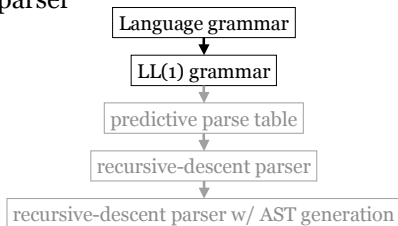
```
Expr parse_S() {
    Expr result = parse_E();
    while (true) {
        switch (token) {
            case '+': token = input.read();
                    result = new Add(result, parse_E());
                    break;
            case ')': case EOF: return result;
            default: throw new ParseError();
        }
    }
}
```

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## Summary

- Now have complete recipe for building a parser



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## Bottom-up parsing

- A more powerful parsing technology
- LR grammars -- more expressive than LL
  - can handle left-recursive grammars, virtually all programming languages
  - Easier to express programming language syntax
- Shift-reduce parsers
  - construct right-most derivation of program
  - automatic parser generators (e.g. yacc, CUP)
  - detect errors as soon as possible
  - allows better error recovery

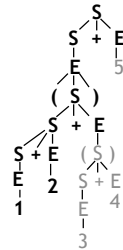
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## Top-down parsing

$(1+2+(3+4))+5$   
 $S \rightarrow S+E \mid E$   
 $E \rightarrow \text{number} \mid (S)$   
 $S \rightarrow S+E \rightarrow E+E \rightarrow (S)+E \rightarrow$   
 $(S+E)+E \rightarrow (S+E+E)+E$   
 $\rightarrow (E+E+E)+E \rightarrow (1+E+E)+E \rightarrow$   
 $(1+2+E)+E \dots$

- In left-most derivation, entire tree above a token (2) has been expanded when encountered
- Must be able to predict productions!



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## Bottom-up parsing

- Right-most derivation -- backward
  - Start with the tokens
  - End with the start symbol

$S \rightarrow S+E \mid E$   
 $E \rightarrow \text{number} \mid (S)$

$(1+2+(3+4))+5 \leftarrow (E+2+(3+4))+5 \leftarrow$   
 $(S+2+(3+4))+5 \leftarrow (S+E+(3+4))+5 \leftarrow$   
 $(S+(3+4))+5 \leftarrow (S+(E+4))+5 \leftarrow (S+(S+4))+5 \leftarrow$   
 $\leftarrow (S+(S+E))+5 \leftarrow (S+(S))+5 \leftarrow (S+E)+5 \leftarrow$   
 $(S)+5 \leftarrow E+5 \leftarrow S+E \leftarrow S$

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## Progress of bottom-up parsing

right-most derivation ↑  
 $(1+2+(3+4))+5 \leftarrow$   
 $(E+2+(3+4))+5 \leftarrow$   
 $(S+2+(3+4))+5 \leftarrow$   
 $(S+E+(3+4))+5 \leftarrow$   
 $(S+(3+4))+5 \leftarrow$   
 $(S+(E+4))+5 \leftarrow$   
 $(S+(S+4))+5 \leftarrow$   
 $(S+(S+E))+5 \leftarrow$   
 $(S+(S))+5 \leftarrow$   
 $(S+E)+5 \leftarrow$   
 $(S)+5 \leftarrow$   
 $E+5 \leftarrow$   
 $S+E \leftarrow$   
 $S$

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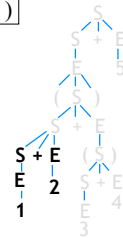
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## Bottom-up parsing

- $(1+2+(3+4))+5 \leftarrow (E+2+(3+4))+5$   
 $\leftarrow (S+2+(3+4))+5$   
 $\leftarrow (S+E+(3+4))+5 \dots$

$S \rightarrow S+E \mid E$   
 $E \rightarrow \text{number} \mid (S)$

- Advantage of bottom-up parsing: can select productions based on more information

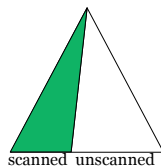


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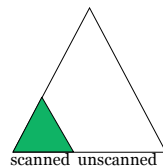
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## Top-down vs. Bottom-up

Bottom-up: Don't need to figure out as much of the parse tree for a given amount of input



Top-down



Bottom-up

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## Shift-reduce parsing

- Parsing is a sequence of *shift* and *reduce* operations
- Parser state is a stack of terminals and non-terminals (grows to the right)
- Unconsumed input is a string of terminals
- Current derivation step is always stack+input

Derivation step	stack	unconsumed input
$(1+2+(3+4))+5 \leftarrow$		$(1+2+(3+4))+5$
$(E+2+(3+4))+5 \leftarrow$	(E	$+2+(3+4))+5$
$(S+2+(3+4))+5 \leftarrow$	(S	$+2+(3+4))+5$
$(S+E+(3+4))+5 \leftarrow$	(S+E	$+(3+4))+5$

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## Shift-reduce parsing

- Parsing is a sequence of *shifts* and *reduces*
- Shift** : move look-ahead tokens to stack

stack	input	action
(	1+2+(3+4))+5	shift 1
(1	+2+(3+4))+5	

- Reduce** : Replace symbols  $\gamma$  in top of stack with non-terminal symbol  $X$ , corresponding to production  $X \rightarrow \gamma$  (pop  $\gamma$ , push  $X$ )

stack	input	action
(S+E	+(3+4))+5	reduce $S \rightarrow S+E$
(S	+(3+4))+5	

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## Shift-reduce parsing

$$\begin{array}{l} S \rightarrow S + E \mid E \\ E \rightarrow \text{number} \mid ( S \end{array}$$

derivation	stack	input stream	action
(1+2+(3+4))+5 ←		(1+2+(3+4))+5	shift
(1+2+(3+4))+5 ←	(	1+2+(3+4))+5	shift
(1+2+(3+4))+5 ←	(1	+2+(3+4))+5	reduce $E \rightarrow \text{num}$
(E+2+(3+4))+5 ←	(E	+2+(3+4))+5	reduce $S \rightarrow E$
(S+2+(3+4))+5 ←	(S	+2+(3+4))+5	shift
(S+2+(3+4))+5 ←	(S+	2+(3+4))+5	shift
(S+2+(3+4))+5 ←	(S+2	+(3+4))+5	reduce $E \rightarrow \text{num}$
(S+E+(3+4))+5 ←	(S+E	+(3+4))+5	reduce $S \rightarrow S+E$
(S+(3+4))+5 ←	(S	+(3+4))+5	shift
(S+(3+4))+5 ←	(S+	(3+4))+5	shift
(S+(3+4))+5 ←	(S+(	3+4))+5	shift
(S+(3+4))+5 ←	(S+(3	+4))+5	reduce $E \rightarrow \text{num}$

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## Problem

- How do we know which action to take -- whether to shift or reduce, and which production?
- Sometimes can reduce but shouldn't
  - e.g.,  $X \rightarrow \epsilon$  can *always* be reduced
- Sometimes can reduce in different ways

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## Action Selection Problem

- Given stack  $\sigma$  and look-ahead symbol  $b$ , should parser:
  - shift**  $b$  onto the stack (making it  $\sigma b$ )
  - reduce** some production  $X \rightarrow \gamma$  assuming that stack has the form  $\alpha \gamma$  (making it  $\alpha X$ )
- If stack has form  $\alpha \gamma$ , should apply reduction  $X \rightarrow \gamma$  (or shift) depending on stack prefix  $\alpha$ 
  - $\alpha$  is different for different possible reductions, since  $\gamma$ 's have different length.
  - How to keep track of possible reductions?

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## Parser States

- Goal: know what reductions are legal at any given point
- Idea: summarize all possible stack prefixes  $\alpha$  as a finite parser *state*
- Parser state is computed by a DFA that reads in the stack  $\alpha$
- Accept states of DFA: unique reduction!
- Summarizing discards information
  - affects what grammars parser handles
  - affects size of DFA (number of states)

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