

**CS 412/413**  
**Introduction to**  
**Compilers and Translators**  
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Lecture 35: Parametric Polymorphism  
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## Polymorphism

- *poly*=many, *morph*=shape
- Code is *polymorphic* if it can be used with values from more than one type
- Have already seen *subtype polymorphism* in Iota+

```
interface set { ... }
class hashSet implements set { ... }
class arraySet implements set { ... }
intersect(s1, s2: set): set
```

- `intersect` works with `s1`, `s2` from *any* class that implements the `set` interface

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## Outline

- Two other forms of polymorphism
  - ad hoc polymorphism (overloading)
  - parametric polymorphism
    - template mechanisms (C++)
    - fully parametric code (ML)
    - bounded parameters (CLU, PolyJ)
- Language design issues
- Type checking
- Code generation
- Appel Chap. 16

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## Overloading

- Overloading: same name can be reused with different types (also: *ad-hoc polymorphism*)
- Ambiguity resolved by static argument types

```
print(int x)
print(string s)
print(float f)
```

print(3)

- Looks like one polymorphic function `print`
  - Reality: three different functions bound to same name—not true polymorphism
- ⇒ Three separate entries in symbol table
- Overloading relies on knowing argument types; conflicts with templates, type inference

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## Parametric polymorphism

- Subtype, ad-hoc polymorphism don't allow abstraction over types
- Example: generic array sort routine  
`sort(a: array[T])` (for all T)
- Type T is a *parameter* to function  
`sort(T: type, a: array[T])`
- *Types* used like values (can pass as arguments). Useful to separate different kinds of arguments:  
`sort[T: type](a: array[T])`

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## Generic array sort

```
sort[T: type](a: array[T]) = (
  i, j: int = 1;
  while (j < n) (
    e: T = a[j];
    i = j-1;
    while (i >= 0 && a[i] > e)
      ( a[i+1] = a[i]; i-- )
    a[i+1] = e
  )
)
```

When do we type check this?

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## Templates (C++)

- Idea: partially evaluate  $\text{sort}(T, a)$  for each distinct  $T$  used in program; type-check partially-evaluated code (*instantiation*)
- ```

sort[int]( a: array[int] ) = (
  i,j:int = 1;
  while (j < n) (
    e:int = a[j];
    i = j-1;
    while (i >= 0 && a[i] > e) ( a[i+1] = a[i]; i-- )
    a[i+1] = e)
  )
  
```
- int > int : ok*
- But: get type-checking errors in library code
  - Libraries must be shipped in source form
  - ">" code differs: must recompile code—bloat!

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## Parametricity

- Can write code in way that doesn't depend on  $T$  using 1<sup>st</sup> class function:
- ```

sort[T: type](a: array[T], gt: function(T,T): bool) = (
  i,j:int = 1;
  while (j < n) (
    e: T = a[j];
    i = j-1;
    while (i >= 0 && gt(a[i], e)) ( a[i+1] = a[i]; i-- )
    a[i+1] = e)
  )
  
```
- α[i] = e*
- Now: code doesn't depend on what  $T$  is: fully *parametric* w/ respect to  $T$
  - Can type-check and generate code *once* for all instantiations (ML)

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## Type checking

- Set of legal type identifiers differs inside a parameterized function

sort[T: type]

(a: array[T], gt: function(T,T): bool) = ( ... )

$T$  in scope as type here

$$A + \{\alpha_1:\text{type}, \dots, \alpha_m:\text{type}\} + \{x_1:T_1, \dots, x_n:T_n\} \vdash e : T_R$$

$$A \vdash f[\alpha_1, \dots, \alpha_m](x_1:T_1, \dots, x_n:T_n) : T_R = e \text{ **defn**}$$

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## Type variables

- Identifier considered a legal type if it appears in the environment

$$\frac{\alpha:\text{type} \in A}{A \vdash \alpha \text{ type}}$$

- No other typing rules mention types  $\alpha$  explicitly

– only rules mentioning  $\frac{id : T \in A}{A \vdash E : T}$  no specific types can apply  $\frac{A \vdash id = E : T}{A \vdash id = E : T}$  (assign)

- ensures static checking works for all actual types used as parameters to code

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## Code generation

- Can generate same code for all instantiations of a parameterized abstraction *if* all types have the same size

```

sort[T: type](a: array[T], gt: function(T,T): bool) = (
  i,j:int = 1;
  while (j < n) (
    e: T = a[j];
    i = j-1;
    while (i >= 0 && gt(a[i], e)) ( a[i+1] = a[i]; i-- )
    a[i+1] = e)
  )
  
```

*How much stack space?*

- Option 1: different code for `sort[int/long]`

- Option 2: auto *box/unbox* to ensure 1-word size

`563836538321` ⇒ `563836538321`

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## Boxing/unboxing

- Idea: some types  $T$  have two different run-time representations

- inline representation

`-double-`

- reference to heap

`•` → `double`

- To support parametric polymorphism: large types (long, double, closures) boxed when used as parameters
- To support primitive type  $T <:$  object, all primitive types boxed when cast to object

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## First-class vs. second-class polymorphic values

sort[T: type] (a: array[T], gt: function(T,T): bool)

- sort[int] has type  
array[int] × (int×int→bool) → unit
- sort is a *polymorphic value*, type  
∀T. array[int] × (T×T→bool) → unit
- Most languages (incl. ML): polymorphic values are in environment but can only be instantiated
- First class polymorphic values: powerful, but can't figure out all instantiations in advance! (must box)

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## Parameterized types

- sort[T: type] : parameterized function that works for any type *T*
- array[T]: parameterized *type* that can be constructed for any type *T*  
array: type → type
- Can a language allow user-defined parameterized types?
- Useful for data structures: Set[T], Map[K,V], Stack[T], Vector[T], Hashtable[K, V]

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## Example

- PolyJ: Java extended w/ parameterized types
- Java 1.2+:  
interface Map {  
  boolean containsKey(Object k);  
  Object get(Object k);  
}
- PolyJ:  
interface Map[Key, Value] {  
  boolean containsKey(Key k);  
  Value get(Key k);  
}  
class HashMap[Key, Value] implements  
  Map[Key, Value] { ... }  
Map[String, int] m = new HashMap[String, int] ...

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## Parameterized types vs. Instantiations

```
class Vector[T] {
  void add(T e);
  T get(int i);
  T set(int i, T e);
}
```

Vector: type→type

```
Vector[int] = class {
  void add(int e);
  int get(int i);
  int set(int i, T e);
}
```

- Parameterized types (*Vector*) are not types; can't have a value of type *Vector* (in PolyJ)  
— Vector x; ... Vector[int] y — x[int]; —
- Instantiations (*Vector[int]*) are types

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## Using parameterized types

```
class Vector {
  void add(Object e);
  Object get(int i);
  Object set(int i, Object e);
}

Vector v;
Animal a;
v.add(a); //unchecked
a = (Animal)v.get(i);

class Vector[T] {
  void add(T e);
  T get(int i);
  T set(int i, T e);
}

Vector[Animal] v;
Animal a;
v.add(a);
a = v.get(i);
```

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## Subtyping relations

- What subtyping relations can hold for instantiation types? —depends on parameterized type
- ```
class Vector[T] {
  void add(T e);
  T get(int i);
}

Elephant <: Animal

Vector[Elephant] <: Vector[Animal]?
Vector[Animal] <: Vector[Elephant]?
```

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## Applying subtyping rules

- `Vector[Elephant] <: Vector[Animal]?`  

```

{
  void add(Elephant e);
  Elephant get(int i);
}
        <:
        {
  void add(Animal e);
  Animal get(int i);
}

```

 Nope: `add`  
 Nope: `get`
- `Vector[Animal] <: Vector[Elephant]?`  
 Nope: `get`
- Rule:
  - Subtyping on instantiation is covariant if type parameters appear only as return values
  - is contravariant if appear only as arguments

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## Constrained parametric polymorphism

- ```

class Set[T] {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if (e.equals(elements[i]))
        return true;
    }
  }
}

```
- oops!*
- `Set[T]` doesn't make sense unless `T` has a notion of equality
  - `SortedSet[T]` : `T` must have a total ordering relation

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## Signature constraints

```

SortedSet[T] where T { int compare(T); } {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if (0 == e.compare(elements[i]))
        return true;
    }
  }
}

```

- Constraint: contract between instantiator and parameterized code
- Can only be instantiated on types w/ `compare`
- `SortedSet` code only uses operations guaranteed to exist on any actual type used in instantiation

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## Subtype constraints?

```

class SortedSet[T] where T <: Comparable {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if (0 == e.compare(elements[i]))
        return true;
    }
  }
}
class Comparable {
  int compare(Comparable x);
}

```

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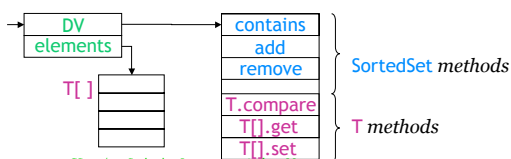
## Implementation

```

SortedSet[T] where T { int compare(T); } {
  ... if (0 == e.compare(elements[i])) ...
}

```

- How to generate code once for all instantiations?  
 Problem: don't know index of `compare` in DV of `T`
- Solution: separate dispatch vector for each instantiation of `SortedSet[T]`



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## Summary

- Overloading is not true polymorphism
- Parametric polymorphism helps write correct code conveniently
- Simple approach: templates. Breaks separate compilation, causes code bloat
- Unconstrained parametric polymorphism: simple to implement, may require boxing/unboxing
- Constrained parametric polymorphism: avoids more run-time errors, can be folded into dispatch vector

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