

Def'n [Command Equivalence]

$$c_1 \sim c_2 \triangleq \forall \sigma, \sigma'. \langle \sigma, c_1 \rangle \Downarrow \sigma' \iff \langle \sigma, c_2 \rangle \Downarrow \sigma'$$

• Reflexive: $c \sim c$.

• Symmetric: $c_1 \sim c_2 \Rightarrow c_2 \sim c_1$.

• Transitive: $c_1 \sim c_2 \wedge c_2 \sim c_3 \Rightarrow c_1 \sim c_3$.

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Prop

c_1 while b do c

\sim

c_2 if b then c; while b do c else skip.

Proof Let σ, σ' be stores.

We'll prove each direction of bi-implication.

(\implies)

Assume $\langle \sigma, c_1 \rangle \Downarrow \sigma'$.

To show: $\langle \sigma, c_2 \rangle \Downarrow \sigma'$.

By inspection, last rule used in big step must have been While-T/F.

Case While-T:

$\langle \sigma, b \rangle \Downarrow \text{true}$

$\langle \sigma, c \rangle \Downarrow \sigma_1$

$\langle \sigma_1, \dots \rangle$

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Prop

$$c_1 \sim c_2 \quad \text{while } b \text{ do } c \quad \sim \quad \text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip.}$$

Proof. Let σ, σ' be stores.

We'll prove each direction of bi-implication.

(\implies)

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Case While-T:

$$\langle \sigma, b \rangle \Downarrow \text{true}$$

$$\langle \sigma, c \rangle \Downarrow \sigma_1$$

$$\langle \sigma_1, \text{while } b \text{ do } c \rangle \Downarrow \sigma'$$



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Prop $\underbrace{\text{while } b \text{ do } c}_{c_1} \sim \underbrace{\text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip.}}_{c_2}$

Proof. Let σ, σ' be stores.

Case While-T:

$$\langle \sigma, b \rangle \Downarrow \text{true}$$

$$\langle \sigma, c \rangle \Downarrow \sigma_1$$

$$\langle \sigma_1, c_1 \rangle \Downarrow \sigma'$$

$$\frac{\langle \sigma, b \rangle \Downarrow \text{true} \quad \frac{\langle \sigma, c \rangle \Downarrow \sigma_1 \quad \langle \sigma_1, c_1 \rangle \Downarrow \sigma'}{\langle \sigma, c; \text{while } b \text{ do } c \rangle \Downarrow \sigma'} \text{Seq}}{\langle \sigma, c_2 \rangle \Downarrow \sigma'} \text{IF-T}$$

✓✓

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(\Leftarrow) Assume $\langle \sigma, c_2 \rangle \Downarrow$

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Prop $\underbrace{\text{while } b \text{ do } c}_{c_1} \sim \underbrace{\text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip.}}_{c_2}$

Proof. Let σ, σ' be stores.

Case while-F:

$$\langle \sigma, b \rangle \Downarrow \text{false}$$

$$\sigma = \sigma'$$

$$\langle \sigma, b \rangle \Downarrow \text{false}$$

$$\frac{}{\langle \sigma, \text{skip} \rangle \Downarrow \sigma} \text{skip}$$

$$\langle \sigma, c_2 \rangle \Downarrow \sigma$$

✓✓

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Proof Let σ, σ' be stores.

(\Leftarrow) Assume $\langle \sigma, c_2 \rangle \Downarrow \sigma'$

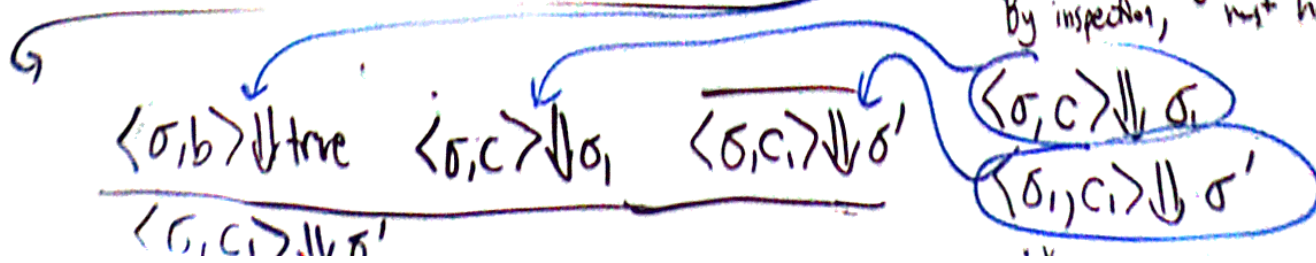
By inspection, last rule used was If-T/F.

Case If-T:

$\langle \sigma, b \rangle \Downarrow \text{true}$

$\langle \sigma, c; c_1 \rangle \Downarrow \sigma'$

By inspection, ? must have used Seq. No.



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if b then c; while b do c else skip.

$c; \text{skip} \sim \text{skip}; c \sim c$

$(c_1; c_2); c_3 \sim c_1; (c_2; c_3)$

$x := a_1; x := a_2 \sim x := a_2$

$x := a; y := b \sim y := b; x := a$
- (assumes $x \neq y$)

Case If-T:

$\langle \sigma, b \rangle \Downarrow \text{true}$

$\langle \sigma, c; c_1 \rangle \Downarrow \sigma'$

spector, ? must have used Seq. No.

$\langle \sigma, c \rangle \Downarrow \sigma_1$

$\langle \sigma_1, c_1 \rangle \Downarrow \sigma'$