

CS 4110

# Programming Languages & Logics

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Lecture 22

Parametric Polymorphism



# Roadmap

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We've extended a simple type system for the  $\lambda$ -calculus with support for a few interesting language constructs. But the “power” of the underlying type system has remained more or less the same.

Today, we'll develop a far more fundamental change to the simply-typed  $\lambda$ -calculus: *parametric polymorphism*, the concept at the heart of OCaml's type system and underlying generics in Java and similar languages.

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- *Ad-hoc polymorphism*, also called overloading, allows the same function name to be used with functions that take different types of parameters.
- *Parametric polymorphism* refers to code that is written without knowledge of the actual type of the arguments; the code is parametric in the type of the parameters.

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$$\text{doubleInt} \triangleq \lambda f: \mathbf{int} \rightarrow \mathbf{int}. \lambda x: \mathbf{int}. f(fx)$$

Now suppose we want the same function for Booleans, or functions...

$$\text{doubleBool} \triangleq \lambda f: \mathbf{bool} \rightarrow \mathbf{bool}. \lambda x: \mathbf{bool}. f(fx)$$

$$\text{doubleFn} \triangleq \lambda f: (\mathbf{int} \rightarrow \mathbf{int}) \rightarrow (\mathbf{int} \rightarrow \mathbf{int}). \lambda x: \mathbf{int} \rightarrow \mathbf{int}. f(fx)$$

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In the doubling functions, the varying parts are the types.

We need a way to abstract out the type of the doubling operation, and later instantiate it with different concrete types.

# Polymorphic $\lambda$ -Calculus

Invented independently in 1972–1974 by a computer scientist John Reynolds and a logician Jean-Yves Girard (who called it System F).

**Key feature:** Function abstraction and application, just like in  $\lambda$ -calculus terms, but *at the type level!*

**Notation:**

- $\Lambda X. e$ : type abstraction
- $e[\tau]$ : type application

**Example:**

$\Lambda X. \lambda x : X. x$

# Polymorphic $\lambda$ -Calculus

## Syntax

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**Typing Judgment:**  $\Delta, \Gamma \vdash e : \tau$

- $\Gamma$  a mapping from variables to types
- $\Delta$  a set of types in scope
- $e$  an expression
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In the polymorphic  $\lambda$ -calculus, however, we can type this expression using a polymorphic type:

$$\vdash \lambda x : \forall X. X \rightarrow X. x [\forall X. X \rightarrow X] x : (\forall X. X \rightarrow X) \rightarrow (\forall X. X \rightarrow X)$$

(However, all expressions in polymorphic  $\lambda$ -calculus still halt. There is no way to give a type to the *self-application* of this term.)

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$$\text{case} \triangleq \Lambda T_1. \Lambda T_2. \Lambda R. \lambda v : T_1 + T_2. \lambda b_1 : T_1 \rightarrow R. \lambda b_2 : T_2 \rightarrow R. \\ v [R] b_1 b_2$$

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# Type Erasure

The following theorem states this translation is adequate:

## Theorem (Erasure Adequacy)

*For all expressions  $e$  and  $e'$ , we have  $e \rightarrow e'$  iff  $\text{erase}(e) \rightarrow \text{erase}(e')$ .*

# Type Inference

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The type inference (or “type reconstruction”) problem asks whether, for a given untyped  $\lambda$ -calculus expression  $e'$  there exists a well-typed System F expression  $e$  such that  $erase(e) = e'$

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See Chapter 23 of Pierce for further discussion, as well as restrictions for which type reconstruction is decidable.