CS 4110

Programming Languages & Logics

Lecture 37
Typed Assembly Language

28 November 2012

Schedule

Last Friday

• Typed Assembly Language

Today

- Polymorphism
- Stack Types

Next Monday

Compilation

TAL-0 Review

- Syntax
- Semantics
- Type System
 - \blacktriangleright Ψ ; $\Gamma \vdash v : \tau$
 - ▶ $\Psi \vdash i : \Gamma \rightarrow \Gamma'$
 - au $au \leq au'$

- ► *H* : Ψ
- **▶** ⊢ *R* : **Γ**
- \vdash (H, R, B)

TAL-0 Review

- Syntax
- Semantics
- Type System

Theorem (Type Safety)

If $\vdash \Sigma$ and $\Sigma \mapsto^* \Sigma'$, then Σ' is not stuck.

Lemma (Progress and Preservation)

- If $\vdash \Sigma_1$ then there exists a Σ_2 such that $\Sigma_1 \mapsto \Sigma_2$
- If $\vdash \Sigma_1$ and $\Sigma_1 \mapsto \Sigma_2$ then $\vdash \Sigma_2$

TAL-1: Polymorphism

Syntax

- Add type variables α and universal types $\forall \alpha. \tau$
- Allow code label types to be polymorphic $\forall \alpha, \beta. \{r_1 : \alpha, r_2 : \beta, r_3 : \{r_1 : \beta, r_2 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}$
- Add type application $v[\tau]$
- Write $v[\tau_1, \ldots, \tau_k]$ for $v[\tau_1] \cdots [\tau_k]$

5

Polymorphism Example

```
swap: \forall \alpha, \beta, \{r_1 : \alpha, r_2 : \beta, r_{31} : \{r_1 : \beta, r_2 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
       mov r_3, r_1
      mov r_1, r_2
       mov r_2, r_3
      imp r_{31}
swap\_ints: \{r_1: int, r_2: int, r_{31}: \{r_1: int, r_2: int\} \rightarrow \{\}\} \rightarrow \{\}
      imp swap [int, int]
swap\_int\_and\_label: \{r_1: int, r_2: \{r_2: int \rightarrow \{\}\}\}
       mov r_{31},L
      imp swap [int, \{r_2: \text{int}\} \rightarrow \{\}]
L: \{r_1: \{r_2: \text{int}\} \to \{\}, r_2: \text{int}\} \to \{\}
      imp r_1
```

Callee-Saves Registers

Common Strategy

- When calling a function...
- Save the contents of some registers on the stack...
- Allow the callee to save (and restore) other designated registers...
- If the callee does not use all registers, the cost of saving and restoring is not incurred...

Correctness Critereon

Callee must return the callee-saves registers to the caller with the same values as when the function was invoked.

Callee-Saves Example

```
callee: \forall \alpha . \{r_1 : \text{int}, r_5 : \alpha, r_{31} : \{r_1 : \text{int}, r_5 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
                                    % Save r_5
      mov r_4, r_5
                                   % Use r_5 for other work
      mov r_5,7
      add r_1, r_1, r_5
                                    % Restore r_5
      mov r_5, r_4
     jmp r_{31}
caller: \{\} \rightarrow \{\}
      mov r_5,255
      mov r_1,5
      mov r_{31},L
     jmp callee[int]
L: \{r_1 : \text{int}, r_5 : \text{int}\} \to \{\}
      mul r_3, r_1, r_5
```

Callee-Saves Bug

```
callee: \forall \alpha . \{r_1 : \text{int}, r_5 : \alpha, r_{31} : \{r_1 : \text{int}, r_5 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
                                   % Save r_5
      mov r_4, r_5
                                   % Use r_5 for other work
      mov r_5,7
      add r_1, r_1, r_5
                                   % Restore r_5
      mov r_5, r_4
     jmp r_{31}
                                   % Error! r_5:int
caller: \{\} \rightarrow \{\}
      mov r_5,255
      mov r_1,5
      mov r_{31},L
     jmp callee[int]
L: \{r_1 : \text{int}, r_5 : \text{int}\} \to \{\}
      mul r_3, r_1, r_5
```

 Can prove that the correct version preserves callee-saves registers

- Can prove that the correct version preserves callee-saves registers
- This follows directly from callee's polymorphic type!

$$\forall \alpha. \begin{cases} r_1 : \text{int,} \\ r_5 : \alpha, \\ r_{31} : \{r_1 : \text{int,} r_5 : \alpha\} \to \{\} \end{cases} \to \{\}$$

- Can prove that the correct version preserves callee-saves registers
- This follows directly from callee's polymorphic type!

$$\forall \alpha. \begin{cases} r_1 : \text{int,} \\ r_5 : \alpha, \\ r_{31} : \{r_1 : \text{int,} r_5 : \alpha\} \to \{\} \end{cases} \to \{\}$$

 Moral: polymorphism is useful for more than just code reuse

- Can prove that the correct version preserves callee-saves registers
- This follows directly from callee's polymorphic type!

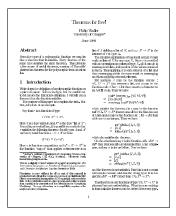
$$\forall \alpha. \begin{cases} r_1 : \text{int,} \\ r_5 : \alpha, \\ r_{31} : \{r_1 : \text{int,} r_5 : \alpha\} \rightarrow \{\} \end{cases} \rightarrow \{\}$$

- Moral: polymorphism is useful for more than just code reuse
- Types can also be used to constrain the behavior of functions

- Can prove that the correct version preserves callee-saves registers
- This follows directly from callee's polymorphic type!

$$\forall \alpha. \begin{cases} r_1 : \text{int,} \\ r_5 : \alpha, \\ r_{31} : \{r_1 : \text{int,} r_5 : \alpha\} \to \{\} \end{cases} \to \{\}$$

- Moral: polymorphism is useful for more than just code reuse
- Types can also be used to constrain the behavior of functions



Paper: P. Wadler. "Theorems for Free!" In *FPCA*, pp. 347–359. September 1989.

We need to make a few small changes to the operational semantics

We need to make a few small changes to the operational semantics

• Heaps *H* now map labels to type-labeled blocks:

$$H(L) = \forall \alpha_1, \ldots, \alpha_k. \Gamma \rightarrow \{\}. B$$

We need to make a few small changes to the operational semantics

• Heaps *H* now map labels to type-labeled blocks:

$$H(L) = \forall \alpha_1, \ldots, \alpha_k. \Gamma \rightarrow \{\}. B$$

• Type variables may appear free in both Γ and B

We need to make a few small changes to the operational semantics

• Heaps *H* now map labels to type-labeled blocks:

$$H(L) = \forall \alpha_1, \ldots, \alpha_k. \Gamma \rightarrow \{\}. B$$

- Type variables may appear free in both Γ and B
- Control-flow operations substitute types

We need to make a few small changes to the operational semantics

• Heaps *H* now map labels to type-labeled blocks:

$$H(L) = \forall \alpha_1, \ldots, \alpha_k. \Gamma \rightarrow \{\}. B$$

- Type variables may appear free in both Γ and B
- Control-flow operations substitute types

$$(H, R, \text{jmp } v[\tau_1, \dots, \tau_k]) \mapsto (H, R, B[\tau_1/\alpha_1, \dots, \tau_k/\alpha_k])$$

where $R(v) = L$ and $H(L) = \forall \alpha_1, \dots, \alpha_k$. $\Gamma \rightarrow \{\}$. B

1

We need to make a few small changes to the operational semantics

• Heaps *H* now map labels to type-labeled blocks:

$$H(L) = \forall \alpha_1, \ldots, \alpha_k. \Gamma \rightarrow \{\}. B$$

- Type variables may appear free in both Γ and B
- Control-flow operations substitute types

$$(H, R, \text{jmp } v[\tau_1, \dots, \tau_k]) \mapsto (H, R, B[\tau_1/\alpha_1, \dots, \tau_k/\alpha_k])$$

where $R(v) = L$ and $H(L) = \forall \alpha_1, \dots, \alpha_k$. $\Gamma \rightarrow \{\}$. B

$$(H, R, \text{beq } r, v[\tau_1, \dots, \tau_k]; B) \mapsto (H, R, B'[\tau_1/\alpha_1, \dots, \tau_k/\alpha_k])$$

where $R(r) = 0, R(v) = L$, and $H(L) = \forall \alpha_1, \dots, \alpha_k. \Gamma \rightarrow \{\}. B'$

Typing Polymormphism

Ψ; Δ; $Γ \vdash v : τ$

Typing Polymormphism

$$|\Psi;\Delta;\Gamma\vdash v:\tau|$$

Type application

$$\frac{\Psi; \Delta; \Gamma \vdash \nu : \forall \alpha_1, \dots, \alpha_k. \Gamma' \to \{\} \qquad \Delta \vdash \tau}{\Psi; \Delta; \Gamma \vdash \nu [\tau] : (\forall \alpha_2, \dots, \alpha_k. \Gamma' \to \{\})[\tau/\alpha]}$$

TAL-2: Stack Types

Run-Time Stack

Almost every compiler uses a run-time stack

- What is a stack?
- A consecutive sequence of memory addresses with one end designated as the top of the stack
- Values are stored on the top of the stack and retrieved later
- The compiler may grow or shrink the stack as needed

Stack uses

- Local variables
- Spilled registers
- Return addresses

Stack Syntax

• Machine states:

$$M ::= (H, R, S, B)$$

• Stacks:

$$S ::= [] | v :: S$$

• Instructions:

$$i ::= \cdots \mid \text{salloc } n \mid \text{sfree } n \mid \text{sld } r_d, n \mid \text{sst } v, n$$

- Errors:
 - Free too many values
 - ► Read too deep in the stack
 - Write too deep in the stack

Stack Instructions

The new stack instructions can be easily encoded:

- A designated register *sp* points to the top of the stack
- salloc *n* subtracts *n* from *sp* (i.e., sub *sp*, *sp*, *n*)
- sfree *n* adds *n* to *sp* (i.e., add *sp*, *sp*, *n*)
- sld r_d , n reads a value at offset n relative to sp (i.e., ld r_d , sp(n))
- sst v, n writes a value at offset n relative to sp (i.e., st sp(n), v)

CISC-like stack instructions can also be encoded:

- push v is salloc 1; sst v, 1
- pop r_d is sld r_d , 1; sfree 1

Example: Factorial

```
fact(n) =
if n \le 0 then 1
else n \times fact(n-1)
```

Example: Factorial

```
fact:
       bgt r_1,L1
                         % if n > 0, goto L1
       mov r_1,1
                         % if n < 0, return
       jmp r_{31}
L1:
       salloc 2
                         % allocate space for frame
                         % save return address
       sst r_{31},1
       sst r_1.2
                         % save n
       sub r_1, r_1, 1
                         % n := n - 1
                         % set return address
       mov r_{31},L
       jmp fact
                         % recursive call
     sld r_2,2
                         % restore n
                         % restore return address
       sld r_{31}, 1
       sfree 2
                         % free space for frame
                         % result := n \times fact(n-1)
       mul r_1, r_1, r_2
                         % return
       jmp r_{31}
```

$$(H, R, S, \text{salloc } n; B) \mapsto (H, R, ? :: \cdots :: ? :: S, B)$$

$$(H, R, S, \text{salloc } n; B) \mapsto (H, R, ? :: \cdots :: ? :: S, B)$$

$$(H, R, v_1 :: \cdots :: v_n :: S, \text{ sfree } n; B) \mapsto (H, R, S, B)$$

$$\overline{(H,R,S,\text{salloc }n;B)} \mapsto (H,R,?::\cdots::?::S,B)$$

$$\overline{(H,R,v_1::\cdots::v_n::S,\text{sfree }n;B)} \mapsto (H,R,S,B)$$

$$S = v_1::\cdots::v_n::S'$$

$$\overline{(H,R,S,\text{sld }r_d,n;B)} \mapsto (H,R[r_d:=v_n],S,B)$$

$$\overline{(H,R,S,\operatorname{salloc} n;B)} \mapsto (H,R,?::\cdots::?::S,B)$$

$$\overline{(H,R,v_1::\cdots::v_n::S,\operatorname{sfree} n;B)} \mapsto (H,R,S,B)$$

$$S = v_1::\cdots::v_n::S'$$

$$\overline{(H,R,S,\operatorname{sld} r_d,n;B)} \mapsto (H,R[r_d:=v_n],S,B)$$

$$S = v_1::\cdots::v_n::S'$$

$$\overline{(H,R,S,\operatorname{sst} v,n;B)} \mapsto (H,R,v_1::\cdots::R(v)::S',B)$$

Type System

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

Type System

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

• [] represents empty stacks

Type System

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

- [] represents empty stacks
- τ :: σ represents stacks with top of type τ and rest of type σ

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

- [] represents empty stacks
- $au :: \sigma$ represents stacks with top of type au and rest of type σ
- ullet ho is a stack type variable

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

- [] represents empty stacks
- τ :: σ represents stacks with top of type τ and rest of type σ
- ρ is a stack type variable
- Register file types contain a special variable sp

```
\{sp : int :: int :: [], r_1 : int, ...\}
```

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

- [] represents empty stacks
- τ :: σ represents stacks with top of type τ and rest of type σ
- ρ is a stack type variable
- Register file types contain a special variable sp

$$\{sp : int :: int :: [], r_1 : int, ...\}$$

Code label types can be polymorphic over stack types

$$\forall \rho. \{ sp : \text{int} :: \rho, r_1 : \text{int} \} \rightarrow \{ \}$$

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

- [] represents empty stacks
- τ :: σ represents stacks with top of type τ and rest of type σ
- ρ is a stack type variable
- Register file types contain a special variable sp

$$\{sp : int :: int :: [], r_1 : int, ... \}$$

Code label types can be polymorphic over stack types

$$\forall \rho. \{ sp : int :: \rho, r_1 : int \} \rightarrow \{ \}$$

Junk values "?" have junk types "?"

$$\boxed{\Psi;\Delta\vdash i:\Gamma_1\to\Gamma_2}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack allocation

$$\frac{\Gamma(sp) = \sigma}{\Psi; \Delta \vdash \mathsf{salloc} \, n : \Gamma \to \Gamma[sp :=? :: \cdots ::? :: \sigma]}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack allocation

$$\frac{\Gamma(sp) = \sigma}{\Psi; \Delta \vdash \mathsf{salloc} \ n : \Gamma \to \Gamma[sp :=? :: \cdots ::? :: \sigma]}$$

Stack free

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \text{sfree } n : \Gamma \to \Gamma[sp := \sigma]}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack load

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \mathsf{sld} \ r_d, n : \Gamma \to \Gamma[r_d := \tau_n]}$$

$$|\Psi; \Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2|$$

Stack load

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \mathsf{sld} \, r_d, n : \Gamma \to \Gamma[r_d := \tau_n]}$$

Stack store

$$\frac{\Psi; \Delta; \Gamma \vdash v : \tau \qquad \Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash sst v, n : \Gamma \rightarrow \Gamma[sp := \tau_1 :: \cdots :: \tau :: \sigma]}$$

Example: Factorial Bug

```
fact: \forall \rho. \{sp : \rho, r_1 : int, r_{31} : \{r_1 : int, sp : \rho\} \rightarrow \{\}\} \rightarrow \{\}
      bgt r_1, L1[\rho]
      mov r_1, 1
      jmp r_{31}
L1: \forall \rho. {sp: \rho, r₁: int, r₃₁: {r₁: int, sp: \rho} → {}} → {}
      salloc 2
      sst r_{31},1
      sst r_1,2
      sub r_1, r_1, 1
      mov r_{31},L[\rho]
      imp fact
L: \forall \rho. \{ sp : \{ r_1 : int, sp : \rho \} \rightarrow \{ \} :: int :: \rho, r_1 : int \} \rightarrow \{ \}
      sld r_2.2
      sld r_{31}, 1
      sfree 2
      \text{mul } r_1, r_1, r_2
                                 % Error! sp: \{r_1: int, sp: \rho\} \rightarrow \{\}:: int:: \rho
      jmp r_{31}
```

Example: Callee Bug

```
caller: \forall \rho. {sp : \tau_{code} :: \rho} \rightarrow {}
       salloc 1
       mov r_1, 17
       sst r_1,1
       mov r_{31},L[\rho]
      imp callee[\tau_{code} :: \rho]
callee: \forall \rho. {sp : int :: \rho, r_{31} : \{sp : \rho, r_1 : int\} \rightarrow \{\}\} \rightarrow \{\}
       sld r_1,1
       add r_1, r_1, r_1
       sst r_1,2
                                                 % Error!
       sfree 1
      jmp r_{31}
L: \forall \rho. \{ sp : \tau_{code} :: \rho, r_1 : int \} \rightarrow \{ \}
```

Type Safety

- Type safety ensures we don't get stuck
- With a few additional features, can handle exceptions
- Paper: G. Morrisett, K. Crary, N. Glew, and D. Walker.
 "Stack-based Typed Assembly Language." In JFP. 12(1):43–88.
 January 2002.

J. Functional Programming 12 (1): 43-88. January 2002. © 2002 Cambridge University Press.

DOI: 10.1017/50956796801004178 Pricted in the United Kinedom

Stack-based typed assembly language

GREG MORRISETT*
Department of Computer Science, Cornell Universit
Black, NY 14853, USA

KARL CRARY
Computer Science Department, Carnegie Mellon University

NEAL GLEW Intertrust, 4750 Patrick Henry Drice, Samu Clara,

DAVID WALKER

Computer Science Department, Carnegie Mellon University 5000 Forbes Acessus, Phtsburgh, PA 15213, USA

Abstract

This paper process STALs a varies of Typed Anomaby Language with constructs and pages to support a similar form of such allocations. As with onder statistically-spill newbord pages to support a similar of the such as the s

Cansule Review

The ability to type-check low-level cusculable code plays an importunt role in counting and execution of untracted code in a secure continuum, each are Mappine, malitic code, and uner-possided kernel extensions. Bytecode verification in fave is a well-known example of type-checking excentible code, but it appears only to a specific tharth high-level virindarian machine interaction set. Typed Amenthy Language (TAL), introduced by Morrison of 10%, extend this supposals to much low-order descrutable code in procedule supersystem for a language similar to the machine code of contemporary processors. However, one limitation of TAL is that it applies only to code compelal in continuum-possing spite, that is

*This material is based on work supported in part by the APOSR grast F40630-97-1-0013, ARDN, RADC grast F3060-96-1-0017, ARDN, RADC grast F3060-96-1-0007, AASERT grast N50014-91-0008, ARDR grast F105240-96-0093, and S507-1-0008, ARDR grast F105240-96-0093.
AND STATE GRAST GRAST

• Note that we didn't bake in a specific calling convention

- Note that we didn't bake in a specific calling convention
- Stacks plus jmp were sufficient

- Note that we didn't bake in a specific calling convention
- Stacks plus jmp were sufficient
- Easy to handle tail calls

- Note that we didn't bake in a specific calling convention
- Stacks plus jmp were sufficient
- Easy to handle tail calls
- Polymorphism is critical

- Note that we didn't bake in a specific calling convention
- Stacks plus jmp were sufficient
- Easy to handle tail calls
- Polymorphism is critical
- Can encode many calling conventions:
 - Arguments on stack or in registers?
 - ► Results on stack or in registers?
 - Return address: caller pops? callee pops?
 - ► Registers: caller saves? callee saves?

- Note that we didn't bake in a specific calling convention
- Stacks plus jmp were sufficient
- Easy to handle tail calls
- Polymorphism is critical
- Can encode many calling conventions:
 - Arguments on stack or in registers?
 - Results on stack or in registers?
 - Return address: caller pops? callee pops?
 - ► Registers: caller saves? callee saves?

Moral: orthogonal combination of type system constructs makes it easy to scale language features