Stochastic Simulations

Six degrees of Kevin Bacon

Outline

- Announcements:
  - Homework II: due Today, by 5, by e-mail
    - Discuss on Friday.
  - Homework III: on web
- Cookie Challenge
- Monte Carlo Simulations
- Random Numbers
- Example--Small Worlds

Homework III

- For HW IV--you will create your own programming assignment.
  - Develop a solution (function or two) to a particular problem
  - Pick something relevant to you!
- For HW III
  - Define your problem
  - Will give me a chance to comment
Monte Carlo

- Monte Carlo methods refer to any procedure that uses random numbers
- Monte Carlo methods are inherently statistical (probabilistic)
- Used in every field
  - Galaxy formation
  - Population model
  - Economics
  - Computer algorithms

Monte Carlo Example

- Have a computer model which computes price of corn in Omaha using rainfall.
  - You have a forecast of rainfall for next few months from NWS
  - Forecast is rain +/- SE
  
  \[
  \begin{align*}
  \text{rain}_1 & \to \text{Model} & \to \$_1 \\
  \text{rain}_2 & \to \text{Model} & \to \$_2 \\
  \text{rain}_3 & \to \text{Model} & \to \$_3
  \end{align*}
  \]

- How can you incorporate uncertainty of rainfall into your forecast of prices?
  - Want $ +/- SE

Monte Carlo Example

1. Create several random forecast rainfall series
   - mean of the series is the forecast
   - SE of series is the forecast SE
2. Compute prices
3. Calculate SE of prices.
Random Numbers

- Computers are deterministic
  - Therefore, computers generate "pseudo-random" numbers
- Matlab’s random numbers are "good"
  - "The uniform random number generator in MATLAB 5 uses a lagged Fibonacci generator, with a cache of 32 floating point numbers, combined with a shift register random integer generator."

Random functions

- rand(m,n) produces m-by-n matrix of uniformly distributed random numbers [0,1]
- randn(m,n) produces random numbers normally distributed with mean=0 and std=1
- randperm(n) is a random permutation of integers [1:n]
  - I=randperm(n); B=A(I,:) scrambles the rows of A

Seeds

- Random number generators are usually recurrence equations:
  - r(n)=F(r(n-1))
- Must provide an initial value r(0)
  - Matlab’s random functions are seeded at startup, but THE SEED IS THE SAME EVERY TIME!
  - Initialize seed with rand('state', sum(100*clock ))
  - How would you ensure rand is always random?
Monte Carlo Example

1. Create several random forecast rainfall series
   - rain, rainerr: n-by-1 vectors of rain forecasts and SE
     - P = randn(n,p)
     - randrain = P*(rainerr*ones(1,p))+(rain*ones(1,p))
2. Compute prices
   - for j = 1:p; prices(:,j) = Model(randrain(:,j)); end
3. Calculate SE of prices.
   - priceerr = std(prices,2)/sqrt(p);
   - pricemn = mean(prices,2);

It's a Small, Small World

- Complicated systems can be viewed as graphs
  - describe how components are connected

Example: Six Degrees of Kevin Bacon

- Components (vertices) are actors
- Connections (edges) are movies
- Hypothesis: 6 or fewer links separate Kevin Bacon from all other actors.
  - "Oracle of Bacon" at http://www.cs.virginia.edu/oracle/
Example: Kevin Bacon & Gollum

Other Systems
- Power Grid
- Food Webs
- Nervous system of Caenorhabditis elegans
- Goal is to learn about these systems by studying their graphs
- Many of these systems are "Small Worlds"--only a few links separate any two points

Watts & Strogatz
- Can organize graphs on a spectrum from ordered to random
- How do graph properties change across this spectrum?
  - $L=$mean path length (# links between points)
  - $C=$cluster coefficient ("lumpiness")
- Used a Monte-Carlo approach--created lots of graphs along spectrum and computed $L$ and $C$
Watts & Strogatz

- Creating the graphs
- \( n \) = # of vertices, \( k \) = number of edges/vertex
- Start with a regular ring lattice and change edges at random with probability \( p \)
- For every \( p \), compute stats for many graphs

Small Worlds in Matlab

- \( G = \text{create lattice}(n,k,p) \)
  - creates a lattice—represented as a sparse matrix
- \([L,C] = \text{latticestats}(G)\)
  - computes the path length and clustering stats
- \([L,C] = \text{Small WorldsEx}(n,k,P,N)\)
  - Creates \( N \) graphs for every \( P(j) \) and saves the mean stats in \( L(j) \) and \( C(j) \)
- \( \text{plot lattice}(G) \)
  - Plots a lattice