CS3810 Assignment 3
Due: Friday, September 17 (in class)

• Write your NetId on every page you turn in!
• Submit each problem on a separate page.
• You may work with other people, but write your solution in your own words and understand everything you turn in. Make sure to justify your answers!

1. (10 pts) Interweaving, Take Two: Define two languages
   \( L_1 = \{a^n b^n | n \geq 1\}^* \) and
   \( L_2 = \{a^n b^n | n \geq 1\}^* \cap a^* \{b^n a^n | n \geq 1\}^* b^* \).
   
   (a) Give all strings in \( L_1 \) of length less than or equal to 6.
   (b) Give all strings in \( L_2 \) of length less than or equal to 8.
   (c) Explain in words the difference between the two languages. If any of the strings you gave above are in one set but not the other, show how they are excluded from the other set.

2. (20 pts) Pumping Lemma: Prove the following languages are not regular:
   
   (a) \( L = \{0^n 1^m | m \geq n\} \)
   (b) \( L = \{0^n 1^m 2^n | n \geq 1\} \)
   (c) \( \{1^n w | n \geq 1, w \text{ is a string of 0's and 1's with at most } n \text{ 1's}\} \)
   (d) The set of strings of zeros and ones whose length is a perfect square.
   (e) The set of strings of \( a \)'s and \( b \)'s of the form \( ww^R \), where \( w^R \) is the reverse of the string \( w \); eg, \( w = aababba \Rightarrow w^R = abbbabaa. \)
   (f) \( \{a^n | n \text{ not prime}\} \)

3. (20 pts) Closure Properties
   
   (a) Min: For a language \( A \) defined over the alphabet \( \{0,1\} \), define \( \text{min}(L) \) as
      \( \text{min}(L) = \{w | w \text{ is in } L, \text{ but no proper prefix of } w \text{ is in } L\} \). Are the regular languages closed under \( \text{min}\)? Prove your answer.
   (b) For any regular language \( L \), define the language \( \text{forward - reverse} \) as the set of strings of \( a \)'s and \( b \)'s of the form \( ww^R \), where \( w \in L \) and \( w^R \) is the reverse of the string \( w \); eg, \( w = aababba \Rightarrow w^R = abbbabaa. \) Is the set of regular languages closed under forward-reverse? Prove your answer.
   (c) Given two strings \( w, x \) of length \( 2k \) such that \( w = a_1 a_2 ... a_{2k} \) and \( x = b_1 b_2 ... b_{2k} \), define a double-alternation of \( w, x \) as \( \text{double - alt}(w, x) = a_1 a_2 b_1 b_2 ... a_{2k-1} b_{2k-1} a_{2k} b_{2k} \). Given two languages \( A \) and \( B \), define \( \text{double - alt}(A, B) \) as \( \text{double - alt}(A, B) = \{ \text{double - alt}(w, x) | w \in A, x \in B, |w| = |x| = 2k, k \geq 1\} \); that is, it is the set of all possible double-alternations (note that for \( \text{double - alt}(w, x) \) to be a valid double-alternation, the lengths of \( w \) and \( x \) must be equal and even.) Is the set of regular languages closed under double alternation? Prove your answer.
   (d) (10 pts) Given a language \( L \), define \( \text{first-two-thirds}(L) \) as \( \text{first-two-thirds}(L) = \{w | \text{there exists an } x \text{ such that } |w| = 2|x|, wx \in L\} \). Are the regular languages closed under first-two-thirds?

4. (10 pts) Converting FAs to REs Convert the following finite automata to regular expressions: (don’t worry—much more manageable than last week’s!)

(a) ![Automaton](image)
(b) ![Automaton](image)