CS381 First Mid Term Solutions Fall 2005

1. Let $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ be finite automata. Construct a finite automaton M that accepts the language $L_1(M_1) \cap L_2(M_2)$.

$M = (Q_1 \times Q_2, \Sigma, \delta, F_1 \times F_2)$ where $\delta([q,p],a) = [\delta(q,a), \delta(p,a)]$

2. Write a regular expression denoting the set of all strings of 0's and 1's such that every 1 is immediately preceded by at least two consecutive 0's.
Suggestion: To make sure your answer is correct ask your self some simple questions such as does your expression work for strings with an even number of 1's, an odd number of 1's, short strings, etc.

$(0^*001)^* 0^*$

3. Let $L_1 = \{ww \mid w \ni (0+1)^*\}$ and $L_2 = \{0^n 1^n \mid n \geq 1\}$. Express L_2 in terms of L_1 using homomorphisms, inverse homomorphisms and intersection with regular sets.

One possibility is
$L_2 = h_2\left(h_1^{-1}(L_1 \cap 0^*10^*1) \cap 0^*1\hat{0}^*1\right)$ where $h_1(0) = 0, h_1(\hat{0}) = 0, h_1(1) = 1$ and $h_2(0) = 0, h_2(\hat{0}) = 1, h_2(1) = \varepsilon$. Note $L_1 \cap 0^*10^*1 = \{0^n 1 0^n 1 \mid n \geq 1\}$ and $h_1^{-1}(L_1 \cap 0^*10^*1) \cap 0^*1\hat{0}^*1 = \{0^n 1 \hat{0}^n 1 \mid n \geq 1\}$