CS381 Solution to First Mid Term Friday Oct 1, 2004
Fall 2004 Hollister B14 9:05-9:55

First Mid Term Solutions

1. Write a regular expression denoting all strings in which every third symbol is a 0. Some strings in the set are ε, 010, 1101101, 0001101001, etc

$$\big((0+1)(0+1)0\big)^*\big(\varepsilon+0+1+(0+1)(0+1)\big)$$

2. Express the set

$$\left\{0^n10^{n-1}10^{n-2}1\cdots1000100101 \mid n\ge 1\right\}$$

in terms of intersection, $\cup, \bullet,$ and $*$ and the set $\left\{0^{i+1}10^i1 \mid i\ge 1\right\}$.

$$\left\{0^{i+1}10^i1 \mid i\ge 1\right\}^*(\varepsilon+01)\cap 0^*1\left\{0^{i+1}10^i1 \mid i\ge 1\right\}(\varepsilon+01)$$

3. Use the pumping lemma to prove that $L=\left\{a^ib^j \mid i\le j\right\}$ is not regular.

Assume that $L=\left\{a^ib^j \mid i\le j\right\}$ is regular and let n be the integer of the pumping lemma. Select the string $w=a^nb^n$. Then w can be written xyz where $|xy|\le n$ and xy^iz is in the set L for all i. Since $|xy|\le n$, y must consist of all a's. Thus xy^2z has more a's than b's and hence is not in the set L, a contradiction. Thus our assumption that the set L was regular is false. We conclude that L is not a regular set.

4. Use homomorphism, inverse homomorphisms and intersection with regular sets to express the set obtained from an arbitrary set L by deleting in each string every 1 appearing in an even numbered position and preceded by a 0.

Let $h_1(0)=0,\ h_1(1)=1,\ h_1(\hat{1})=1$. Let R be the regular set $(00+0\hat{1}+10+11)^*(\varepsilon+0+1)$. Let $h_2(0)=0,\ h_2(1)=1,\ h_2(\hat{1})=\varepsilon$. The desired set is $h_2\left(h_1^{-1}(L)\cap R\right)$.