First Mid Term Solutions

1. Write a regular expression denoting all strings in which every third symbol is a 0. Some strings in the set are $\varepsilon$, 010, 1101101, 0001101001, etc

$((0+1)(0+1)0)^*\varepsilon + 0 + 1 + (0+1)(0+1)$

2. Express the set

$\left\{0^n10^{n-1}10^{n-2}1\cdots 1000100101 | n \geq 1\right\}$

in terms of intersection, $\cup$, $\cdot$, and $\ast$ and the set $\left\{0^{i+1}10^i | i \geq 1\right\}$.

$\left\{0^{i+1}10^i | i \geq 1\right\}\ast \varepsilon \cup 0\ast 1\left\{0^{i+1}10^i | i \geq 1\right\} (\varepsilon + 01)$

3. Use the pumping lemma to prove that $L = \left\{a^ib^j | i \leq j\right\}$ is not regular.

Assume that $L = \left\{a^ib^j | i \leq j\right\}$ is regular and let $n$ be the integer of the pumping lemma. Select the string $w = a^n b^n$. Then $w$ can be written $xyz$ where $|xy| \leq n$ and $xy^iz$ is in the set $L$ for all $i$. Since $|xy| \leq n$, $y$ must consist of all $a$’s. Thus $xy^2z$ has more $a$’s than $b$’s and hence is not in the set $L$, a contradiction. Thus our assumption that the set $L$ was regular is false. We conclude that $L$ is not a regular set.

4. Use homomorphism, inverse homomorphisms and intersection with regular sets to express the set obtained from an arbitrary set $L$ by deleting in each string every 1 appearing in an even numbered position and preceded by a 0.
Let $h_1(0) = 0$, $h_1(1) = 1$, $h_1(\hat{1}) = 1$. Let $R$ be the regular set $(00 + 0\hat{1} + 10 + 11)^* (\varepsilon + 0 + 1)$. Let $h_2(0) = 0$, $h_2(1) = 1$, $h_2(\hat{1}) = \varepsilon$. The desired set is $h_2(h_1^{-1}(L) \cap R)$. 