Problem 1. Let $L = \{1010010001\ldots10^n \mid n \geq 0\}$, that is, it has strings which, for some nonnegative $n$, have $n$ blocks of zeros, separated by single 1’s and each successive block one longer than the previous one, starting with length 1. Construct CFGs $G_1$ and $G_2$ such that $L(G_1) \cap L(G_2) = L$.

Problem 2. Design a PDA that accepts the all strings of a’s and b’s in which no prefix has more b’s than a’s. You may use acceptance by final state or empty pushdown store, whichever you find more convenient.

Problem 3. Give a convincing argument that the class of CFLs is closed under the operations of union, concatenation and star.

Problem 4. Give a machine construction to show that if $L_1$ and $L_2$ are languages accepted by PDAs, then $L_1 \cup L_2$ is also accepted by a PDA.

Problem 5. What goes wrong with your construction when you try to do Problem 4 for intersection rather than union?

The following problem is a problem to think about and discuss with your classmates. This is not a homework problem.

Extra Problem. A CFG is ambiguous if some sentence has two or more left most derivations. A CFL is inherently ambiguous if every CFG generating it is ambiguous. Can you find an unambiguous grammar generating the following set

\[ L = \{a^n b^n c^m d^n \mid n, m \geq 0\} \cup \{a^n b^m c^m d^n \mid n, m \geq 0\}. \]