Problem 1. Describe the set
\[ \{0^n10^{n+1} | n \geq 1\}^* \cap 01\{0^n10^{n+1} | n \geq 1\}^*0^*1. \]
Are there any strings with an odd number of blocks of zeros?

Problem 2. (Problem 2.2.6(a)) Give a DFA accepting the following language over alphabet \{0,1\}:
The set of all strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings 101, 1010, and 1111 are in the language; 0, 100, and 111 are not.

Problem 3. (Problem 2.2.10) Consider the DFA with the following transition table:

\[
\begin{array}{c|cc}
& 0 & 1 \\
\hline
\rightarrow & A & A \\
B & B & A
\end{array}
\]
Informally describe the language accepted by this DFA, and prove by induction on the length of an input string that your description is correct. Hint: When setting up the inductive hypothesis, it is wise to make a statement about what inputs get you to each state, not just what inputs get you the accepting state.

Problem 4. (Problem 2.3.1) Convert to a DFA the following NFA:

\[
\begin{array}{c|cc}
& 0 & 1 \\
\hline
\rightarrow & \{p,q\} & \{p\} \\
qu & \{r\} & \{r\} \\
r & \{s\} & \emptyset \\
\ast s & \{s\} & \{s\}
\end{array}
\]

Problem 5. (Problem 2.3.4) Give nondeterministic finite automata to accept the following languages. Try to take advantage of nondeterminism as much as possible.

(a) The set of strings over alphabet \{0,1,\ldots,9\} such that the final digit has appeared before.
(b) The set of strings over alphabet \{0,1,\ldots,9\} such that the final digit has not appeared before.
(c) The set of strings of 0’s and 1’s such that there are two 0’s separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.

Extra Problem. The following problem is a problem to think about and discuss with your classmates. Not a homework problem.
Which of the following six sets are of the same cardinality?

(a) The set of all finite length strings over the alphabet \{0,1\}.
(b) The set of all finite length strings over the alphabet \{0,1,2,3\}.
(c) The set of all finite length strings of integers.
(d) The set of all functions mapping integers to \{0,1\}.
(e) The set of all functions mapping integers to \{0,1,2,3\}.
(f) The set of all functions mapping integers to integers.