1. Write a regular expression denoting all strings in which every third symbol is a 0. Some strings in the set are $\varepsilon$, 010, 1101101, 0001101001, etc.

$$(0+1)(0+1)0^* (\varepsilon + 0+1 + (0+1)(0+1))$$

2. Express the set
$$\left\{ 0^n10^n-110^n-2\cdots 1000100101 | n \geq 1 \right\}$$
in terms of intersection, $\cup$, $\cdot$, and $*$ and the set $\left\{ 0^{i+1}10^i1 | i \geq 1 \right\}$.

$$\left\{ 0^{i+1}10^i1 | i \geq 1 \right\} * (\varepsilon + 01) \cap 0^*1 \left\{ 0^{i+1}10^i1 | i \geq 1 \right\} * (\varepsilon + 01)$$

3. Use the pumping lemma to prove that $L = \left\{ a^ib^j | i \leq j \right\}$ is not regular.
Assume that $L = \left\{ a^ib^j | i \leq j \right\}$ is regular and let $n$ be the integer of the pumping lemma. Select the string $w = a^nb^n$. Then $w$ can be written $xyz$ where $|xy| \leq n$ and $xy^iz$ is in the set $L$ for all $i$. Since $|xy| \leq n$, $y$ must consist of all a’s. Thus $xy^2z$ has more a’s than b’s and hence is not in the set $L$, a contradiction. Thus our assumption that the set $L$ was regular is false. We conclude that $L$ is not a regular set.

4. Use homomorphism, inverse homomorphisms and intersection with regular sets to express the set obtained from an arbitrary set $L$ by deleting in each string every 1 appearing in an even numbered position and preceded by a 0.
Let $h_1(0) = 0, h_1(1) = 1, h_1(\hat{1}) = 1$. Let $R$ be the regular set 
\[(00 + 0\hat{1} + 10 + 11)^*(\varepsilon + 0 + 1)\]. Let $h_2(0) = 0, h_2(1) = 1, h_2(\hat{1}) = \varepsilon$. The desired set is $h_2(h_1^{-1}(L) \cap R)$. 