Concepts

Notation
\[ \varepsilon, \{\varepsilon\}, \Phi \]
\[ L_1 \setminus L_2 = \{xy \mid x \in L_1, y \in L_2\} \]
\[ L^* = \{\varepsilon\} \cup L \cup L^2 \cup \cdots \]
\[ 2^S \quad \text{set of all subsets} \]
\[ \{0^n1^n | n \geq 1\} \text{ and } \{0^n1^n | n \geq 1\}^* \]

Concepts
- object and name of object
- finite but arbitrarily large
- countably infinite
- noncountably infinite
- diagonalization
- \( \{0+1\}^* \), the set of all finite length strings is countably infinite
- \( 2^{[0+1]^*} \), the set of all subsets of finite length strings is not countably infinite
- induction
- fa
- nfa
- \( \varepsilon \)-nfa
- \( \varepsilon \)-closure
- regular set
- regular expression

fa
- construct finite automata from simple set description
- construct regular expression from simple set description
- subset construction, convert nfa to dfa
- cross product construction
- convert fa to regular expression \( R_y^i \)
- convert regular expression to finite automata
- closure properties \( \cup, \bullet, \ast, \cap, \text{complement, h, h}^{-1}, L^R \)
- homomorphisms and inverse homomorphisms
- pumping lemma – statement – proof – applications
- decision properties – membership, emptiness, equivalence, finite, cofinite
- state minimization

context-free languages
- definition of cfg
dpa
conversion from pda to cfg and cfg to pda
CNF
pumping lemma
closure properties substitution, $\cup, \cdot, *, \cap R, h, h^{-1}, L^R$
decision properties membership and emptiness
efficient algorithm for membership – dynamic programming

Turing machines
Concepts
diagonalization
recursive set
recursively enumerable set
decidable
Turing machine
computability

More powerful models
multi tape
multi track
nondeterministic

Weaker models
semi infinite tape
two pushdown store
4-counter machine
2-counter machine $2^\cdot2^k7^l$

$L_D$
halting problem
class of recursive sets closed under complement
class of r.e. sets not closed under complement
listing strings in r.e. set
If $L$ and $\overline{L}$ are both r.e. then $L$ and $\overline{L}$ are both recursive
If $L$ can be enumerated in order, then $L$ is recursive
Can we enumerate names of all recursive sets? (Depends on definition of name.)
Rice’s Theorem: Every nontrivial property on the r.e. sets is undecidable.
concept of reduction

Decidability for cfl’s
set of valid computations of Tm is intersection of two cfl’s
set of invalid computations is a clf
Undecidable
\[ L(G_1) \cap L(G_2) = \Phi \]
\[ L(G) = \Sigma^* \]
\[ L(G_1) = L(G_2) \quad \text{equivalance} \]
\[ L(G_1) \subseteq L(G_2) \]
\[ R \subseteq L(G) \]

Rado’s sigma function

\[ \{ M \mid L(M) \text{ infinite} \} \text{ not r.e. and complement not r.e.} \]

Every r.e. set is the homomorphic image of a recursive set

\textbf{P and NP}

complete problems for NP
\begin{itemize}
  \item 3-CNF satisfiability
  \item clique
  \item Hamilton circuit
\end{itemize}