Concepts

Notation

$\varepsilon, \{\varepsilon\}, \Phi$

$L_1 \square L_2 = \{xy \mid x \in L_1 \ \ y \in L_2\}$

$L* = \{\varepsilon\} \cup L \cup L^2 \cup L^3 \cup \cdots$

2^S set of all subsets

$\{0^n 10^n 1 \mid n \geq 1\}$ and $\{0^n 10^n 1 \mid n \geq 1\}*$

Concepts

object and name of object
finite but arbitrarily large
countably infinite
noncountably infinite
diagonalization
$\{0+1\}*$, the set of all finite length strings is countably infinite
$2^{\{0+1\}*}$, the set of all subsets of finite length strings is not countably infinite
induction
fa
nfa
ε-nfa
ε-closure
regular set
regular expression

fa

construct finite automata from simple set description
construct regular expression from simple set description
subset construction, convert nfa to dfa
cross product construction
convert fa to regular expression R_{ij}^k
convert regular expression to finite automata
closure properties $\cup$, $\bullet$, $*$, $\cap$, complement, h, h^{-1}, L^R
homomorphisms and inverse homomorphisms
pumping lemma – statement – proof – applications
decision properties – membership, emptiness, equivalence, finite, cofinite
state minimization

context-free languages

definition of cfg

dpa
conversion from pda to cfg and cfg to pda
CNF
pumping lemma
closure properties substitution, $\bigcup$, $\bullet$, $*$, $\bigcap R$, h, h^{-1}, L^R
decision properties membership and emptiness
efficient algorithm for membership – dynamic programming

Turing machines
Concepts

diagonalization
recursive set
recursively enumerable set
decidable
Turing machine
computability

More powerful models

multi tape
multi track
nondeterministic

Weaker models

semi infinite tape
two pushdown store
4-counter machine
2-counter machine $2^i 2^j 5^k 7^l$

L_D
halting problem
class of recursive sets closed under complement
class of r.e. sets not closed under complement
listing strings in r.e. set
If L and $\overline{L}$ are both r.e. then L and $\overline{L}$ are both recursive
If L can be enumerated in order, then L is recursive
Can we enumerate names of all recursive sets? (Depends on definition of name.)
Rice's Theorem: Every nontrivial property on the r.e. sets is undecidable.
concept of reduction

Decidability for cfl's

set of valid computations of Tm is intersection of two cfl's
set of invalid computations is a clf

Undecidable

$L(G_1) \bigcap L(G_2) = \Phi$

$L(G) = \Sigma *$

$L(G_1) = L(G_2) \quad \textit{equvilance}$

$L(G_1) \subseteq L(G_2)$

$R \subseteq L(G)$

Rado's sigma function

$\{M \mid L(M) \text{ infinite}\}$ not r.e. and complement not r.e.

Every r.e. set is the homomorphic image of a recursive set

P and NP

- complete problems for NP
  - 3-CNF satisfiability
  - clique
  - Hamilton circuit