COMS 381, Summer 2005
A few guidelines for writing better proofs

The first criterion for a good proof, of course, is that it should be correct. This should ideally be the only criterion I need to take into account while grading, and this is the only thing your readers should need to take into account when they are reading a proof you present in a research paper. With this in mind, it makes sense to write your proofs clearly, so that the reader can get your mathematical message directly and not become confused about the layout, notation or flow of your proof.

Based on my observations while grading your proofs this year, here are my most important pieces of advice for you:

- State your proof objective (what you are trying to prove) at the beginning, and give a conclusion at the end. One sentence at the beginning and one sentence at the end of the proof is enough.

- In most cases, you should state - somewhere close to the beginning of the proof - your proof technique. Again, one sentence is enough. For example, say: "Proof by contradiction", or "Proof by induction on the length of x", or "Proof of nonregularity using the pumping lemma in the demon game scenario".

- In longer proofs - such as the ones in Homework 4, which have several parts - state up front your proof objective and proof technique for the entire proof. For instance "We show that the sets A and $L(G)$ are equal by showing inclusion in both directions. The forward direction will be by induction on the length of x, the backward direction by induction on the length of the derivation". Then, for each subpart (e.g. each direction of an if and only if proof), make it clear which direction you're proving.

- Key steps in your proof (i.e. things your argument really hinges on) should be stated mathematically rather than in English if possible. If you state the crucial points only in English, this leads the reader to believe you are trying to hide something. The pejorative term "handwaving proof" refers precisely to a proof which is not formal enough and therefore not convincing enough.

- Be precise in your terminology. Don't write "represents" if you mean "accepts", don't write "set" if you mean "state", and so on. Yes, I probably know what you meant, but you shouldn't rely on the reader's sympathy to make your meaning clear. Your proofs should be so precise that even a hostile reader has to agree that they're correct!

- Define your notation. If you use standard notation (e.g. x,y,z in the demon game proofs) you don't need to define it again. But if you introduce a new variable, you must define it. If a j or h or A suddenly appears in your proof, it's impossible for me to figure out what it represents. Also please don't switch notation in mid-proof.

- When you make use of a result proved earlier, indicate clearly that the result was proved earlier and where it was proved.