Problem 3. We define a new type of NFA, that we call \( \text{all}-\text{NFA} \). In this, we can have non-deterministic and \( \varepsilon \)-transitions, but now, a string \( w \) is accepted only if \emph{every} path from the start state on \( w \) leads to an accepting state. Prove that the set of languages accepted by \( \text{all}-\text{NFA} \) are exactly the regular languages.

\textbf{Solution.} Let \( L \) be the set of languages accepted by \( \text{all}-\text{NFA} \), and let \( R \) be the set of regular languages. We wish to show that \( L = R \). Notice that a DFA is a special case of an \( \text{all}-\text{NFA} \). Thus, \( R \subseteq L \). To show that \( L \subseteq R \) we will construct a DFA \( D \) accepting the same language as a given \( \text{all}-\text{NFA} \) \( N = (Q, \Sigma, \delta, q_0, F) \). Let \( D = (2^{Q \cup \{q_{\text{trap}}\}}, \Sigma, \delta', \text{ECLOSE}(q_0), 2^F) \), where \( \delta'(S, x) \) is computed as follows (similar to the \( \varepsilon \)-closure construction):

1. Let \( S = \{q_1, ..., q_k\} \).
2. Let \( \delta^*: Q \cup \{q_{\text{trap}}\} \rightarrow 2^{Q \cup \{q_{\text{trap}}\}} \) be defined as follows:
   \[ \delta^*(q, y) = \begin{cases} 
   \delta(q, y) & \text{if } q \in Q \text{ and } \delta(q, y) \neq \varnothing, \\
   \{q_{\text{trap}}\} & \text{otherwise.} 
   \end{cases} \]
3. Compute \( \bigcup_{i=1}^k \delta^*(q_i, x) \); let this set be \( \{r_1, ..., r_m\} \).
4. Then \( \delta'(S, x) = \bigcup_{j=1}^m \text{ECLOSE}(r_j) \).

Intuitively, this is just the \( \varepsilon \)-closure of the \( \text{all}-\text{NFA} \), except that we must keep track of when a path from the start state is “dropped”. This is why we introduce a special state \( q_{\text{trap}} \). Without this addition, our DFA would mistakenly accept if one of the paths was dropped but all the others reached accepting states.

\textit{Common mistakes.}

- Forgetting to account for the “dropped” paths described above (-3)
- Forgetting to argue that \( R \subseteq L \), i.e., only giving the harder direction of the proof (-3)
- Giving an argument for \( L \subseteq R \) that lacks a construction, e.g., claiming that \text{all}-\text{NFA} are “special cases” of regular NFA (-7)

Uncommon, minor mistakes were penalized by 1 or 2 points.