Problem 5  The induction is over the length of the string $x$ that is taken as an input. The inductive hypothesis is

- $\hat{\delta}(A, x) = A$ if $x$ either $\varepsilon$ or $x$ does not have 00 as a substring and ends with 1.
- $\hat{\delta}(A, x) = B$ if $x$ does not have 00 as a substring and ends with 0.
- $\hat{\delta}(A, x) = C$ if $x$ has 00 as a substring.

**Base Case**: Is when $x = \varepsilon$.

**Inductive case**: Assume $x = ya$ and the inductive hypothesis holds for $y$. The three cases are on what the state $\hat{\delta}(A, y)$ is. If this state is $C$, then $x$ has 00 as a substring since $y$ has it. Also, $\hat{\delta}(A, x) = \hat{\delta}(C, a) = C$. If the state $\hat{\delta}(A, y) = B$, then $y = z0$ for some $z$ and so, $x = z0a$. Again by the induction hypothesis, $y$ does not have 00 as a substring, so $x$ contains 00 only if $a = 0$. But in that case, $\hat{\delta}(A, x) = \hat{\delta}(A, y0) = \hat{\delta}(B, 0) = C$. Else, if $a = 1$, $x$ does not have a 00 substring, and ends with a 1 and accordingly, $\hat{\delta}(A, x) = \hat{\delta}(A, y1) = \hat{\delta}(B, 1) = A$. The last remaining case is when $\hat{\delta}(A, y) = A$, and thus $y = z1$ for some $z$. Thus $x$ will not have a 00 substring. Furthermore, depending on whether $a$ is 0 or 1, we have that $\hat{\delta}(A, x) = \delta(A, a)$ to be $A$ or $B$ respectively. Hence we satisfy the inductive hypothesis in all three cases.