Question 2

List the three shortest strings in the set \( \{0^n10^{n+1}|n \geq 1\} \)

1. \( n=1 \): 01001
2. \( n=2 \): 0010001
3. \( n=3 \): 000100001

Describe the set of strings denoted by \( \{0^n10^{n+1}|n \geq 1\}^* \) How many strings are there of length less than or equal to 15?

These are strings with an even number of ones each separated by one or more zeros, such that when each non-overlapping pair of consecutive ones (starting with the first "1" in the string) is considered, each pair has one more zero preceding the second one than the first. Strings with zero pairs of ones (which in this case can only be the empty string because of restrictions above) are also accepted.

This set has 14 elements with length \( \leq 15 \). For this example denote, \( L = \{0^n10^{n+1}|n \geq 1\} \), and \( L_n = \{0^n10^{n+1}\} \).

- First, there is the possibility of \( L^0 = \epsilon \).

- Second, there are 6 elements with length \( \leq 15 \) \( \in L^1 \):

  1. \( |L_1| = 5 \)
  2. \( |L_2| = 7 \)
  3. \( |L_3| = 9 \)
  4. \( |L_4| = 11 \)
  5. \( |L_5| = 13 \)
  6. \( |L_6| = 15 \)

- Now there 6 elements with length \( \leq 15 \) \( \in L^2 \):

  1. \( |L_1L_1| = 10 \)
  2. \( |L_1L_2| = 12 \)
  3. \( |L_1L_3| = 14 \)
  4. \( |L_2L_1| = 12 \)
5. $|L_2 L_2| = 14$
6. $|L_3 L_1| = 14$

• Finally, there is one element with length $\leq 15 \in L^3$.

1. $|L_1 L_1 L_1| = 15$

All together, this is the aforementioned 14.