1. Prove non-regularity of the following languages.
   
   (a) \( L = \{0^i \mid i \geq 0\} \).
   
   (b) \( L = \{w \mid w \in \{0, 1\}^*, w = w^R\} \), where \( w^R \) is the reverse of the string \( w \).
   
   (c) The set PAREN of balanced parenthesis \( () \). For example, \( (()()()) \) belongs to PAREN but \( )())( \) does not.
   
   (d) \( \{0^i w 0^j \mid i, j > 0, w \in \{0, 1\}^*\} \).

2. Prove that if \( A \) is a regular language over \( \Sigma \) and \( \{a, b\} \subseteq \Sigma \), then the following language is also regular.

   \( \{c^n \mid \exists \ w \in A, \#a(w) + \#b(w) = n\} \)

3. 4.2.11.

4. Consider the language \( L = \{a^i b^j c^k \mid i, j, k \geq 0 \land \text{if } i = 1 \text{ then } j = k\} \).
   
   Prove that \( L \) satisfies the conditions of the pumping lemma, i.e. show that there is a number \( p \) where, if \( s \) is a string of length at least \( p \) in \( L \), then \( s \) may be written as \( s = xyz \) such that
   
   \begin{itemize}
   \item for each \( i \geq 0 \), \( xy^i z \in L \).
   \item \( y \neq \varepsilon \).
   \item \( |xy| \leq p \).
   \end{itemize}

   Prove that \( L \) is nonregular. Explain why this fact does not contradict the pumping lemma.