Do each problem on a separate set of pages and please remember to write your name, net-id and problem number on the top right hand corner of each page.

1. 4.2.6 from the book using machine constructions.

2. Let $\mathcal{L}$ be a set of languages such that $\mathcal{L}$ is closed under the following operations:

   (a) If $L_1, L_2$ are in $\mathcal{L}$, so is $L_1 \cdot L_2 = \{xy | x \in L_1, y \in L_2\}$.
   (b) If $L$ is in $\mathcal{L}$, so is $L \cap R$ for any regular language $R$.
   (c) If $L$ is in $\mathcal{L}$, so are $h(L)$ and $h^{-1}(L)$ where $h$ is a homomorphism and $h^{-1}$ is the inverse homomorphism.

   Show that $\mathcal{L}$ is closed under union i.e. if $L_1$ and $L_2$ are in $\mathcal{L}$, then $L_1 \cup L_2$ is too.

3. For $A$ to be a set of natural numbers, define

   \[
   \text{binary}A = \{\text{binary representation of number of } A \} \subseteq \{0, 1\}^* \\
   \text{unary}A = \{\text{unary representation of number of } A \} \subseteq \{0\}^*
   \]

   For example, if $A = \{2, 3, 5\}$, $\text{binary}A = \{10, 11, 101\}$ and $\text{unary}A = \{00, 000, 00000\}$.

   One of the following statements is true and the other is false. State which is which and prove.

   (a) $\forall A$, if $\text{binary}A$ is regular, then $\text{unary}A$ is regular.
   (b) $\forall A$, if $\text{unary}A$ is regular, then $\text{binary}A$ is regular.

4. For this question assume that the following two languages $L_1$ and $L_2$ are not regular.

   \[
   L_1 = \{0^i1^i | i \geq 0\} \\
   L_2 = \{0^i10^i | i \geq 0\}
   \]

   Assuming the closure properties of regular languages, under union, intersection, closure and homomorphisms and its inverse, show that none of the following languages are regular. \textit{Hint}: Show that if they were regular, then $L_1$ or $L_2$ would be regular.
(a) \( \{0^{2i}1^{3i} \mid i \geq 0\} \).
(b) \( \{ww \mid w \in \{0,1\}^*\} \).
(c) \( \{0^i1^j \mid i = j + 50\} \).
(d) \( \{0^i1^j2^k \mid i = j \lor j = k\} \).