1. Do each problem on a separate set of sheets and please remember to write your name, net-id and problem number on each sheet.

2. Suppose the set of input symbols $\Sigma$ contains $k$ elements. Define the language $L$ as follows:

   $$L = \{ w \in \Sigma^* | \exists a, b \in \Sigma, a \neq b, w \text{ does not contain } a \text{ or } b \}$$

   It is easy to define a finite automaton for $L$ having number of states that is exponential in $k$. Define a NFA for $L$ having only $O(k^2)$ states.

3. Let $x$ and $y$ be strings and let $L$ be any language. We say that $x$ and $y$ are distinguishable by $L$ if some string $z$ exists whereby exactly one of the strings $xz$ and $yz$ is a member of $L$. Otherwise, if for every string $z$, $xz \in L$ if and only if $yz \in L$, we say that $x$ and $y$ are indistinguishable by $L$. If $x$ and $y$ are indistinguishable by $L$ we write $x \equiv_L y$.

   (a) Prove that $\equiv_L$ is an equivalence relation i.e. is reflexive, symmetric and transitive.

   (b) Suppose $L$ is a regular language and let $X$ be a set of $k$ strings $\{x_1, \ldots, x_k\}$. Suppose we have that for each pair $i, j \leq k$, such that $i \neq j$, $x_i \not\equiv_L x_j$. Then prove that the DFA that accepts $L$ must have at least $k$ states.

4. Define $L = \{ 0^n1^n | n \geq 0 \}$. Use $L$ along with the operations of union, concatenation, and closure to generate all strings that are not of the form $\{01, 01001, 010010001\ldots\}$ i.e. strings where delimiter 1 separates out consecutive number of zeroes.

5. Problem 4.2.8.