Please write your name and net-id on the upper right corner of every page.

1. 8.4.4.

2. Describe a deterministic Turing machine that accepts the set \( \{ a^{3n} \mid n \geq 0 \} \). Describe the machine informally, in particular, describe the set of transitions and the states in English.

3. A context sensitive grammar is described as \((N, \Sigma, P, S)\) where \(N\) is the set of nonterminals, \(\Sigma\) is the set of terminals, \(S\) is the start symbol and \(P\) are the production rules. The production rules are all of the form 
\[
\alpha \rightarrow \beta
\]
where \(\alpha, \beta \in (N \cup \Sigma)^*\), and \(|\alpha| \leq |\beta|\). Give a context-sensitive grammar for the following languages
(a) \(\{ a^n b^n c^n \mid n \geq 1 \}\). This shows that the context sensitive grammars are strictly more general than context-free ones.
(b) \(\{ a^{n^2} \mid n \geq 1 \}\).

4. A linear bounded automaton (LBA) \(M\) is exactly like a one-tape Turing machine, except that the input string \(x \in \Sigma^*\) is enclosed in a left and right endmarkers \(\downarrow\) and \(\downarrow\) which may not be overwritten, and the machine is constrained never to move to the left of \(\downarrow\) nor to the right of \(\downarrow\). It may read or write all it wants between the endmarkers. The LBA is formally described in terms of a 9-tuple \((Q, \Sigma, \Gamma, \downarrow, \downarrow, \delta, s, t, r)\) where \(s\) is the start state, \(t\) the final state, and \(r\) is a reject state. \(\Sigma\) is the input alphabet and \(\Gamma\) the tape alphabet. \(\delta\) is the transition function.
(a) If \(M\) be an LBA with \(|Q| = k\) and \(|\Gamma| = m\). How many configurations are there on input \(x\), \(|x| = n\).
(b) Prove that for every CSG \(G\), there is a non-deterministic LBA \(M\) such that \(L(M) = L(G)\).
(c) For every nondeterministic LBA \(M\), there is context-sensitive grammar \(G\) such that \(L(G) = L(M) - \{\varepsilon\}\).