Homework 7 Problem 2

Use homomorphisms, inverse homomorphisms, and intersection with regular sets to do the following conversions.

Note: The most common mistake was to apply set intersection with a set that was not regular. It is important to use a regular set since the classes of languages we’re studying are not closed under intersection with arbitrary sets, only regular sets. Also, allowing arbitrary sets trivializes the problem: I can write any set $S$ as $\Sigma^* \cap S$.

For each of the following, assume the original set is called $L$.

(a) Convert $\{a^nb^nc^n \mid n \geq 1\}$ into $\{a^nb^n \mid n \geq 1\}$.

Use a homomorphism to delete every $c$.

$h(a) = a$
$h(b) = b$
$h(c) = \epsilon$

$h(L) = \{a^nb^n \mid n \geq 1\}$

(b) Convert $\{0^i10^{i+1}1 \}$ into $\{0^i10^{2i}1\}$

Use an inverse homomorphism to mark arbitrary 0’s with hats or bars. Use a regular expression to enforce that the first zero in the right block is barred and the rest are hatted. Use another homomorphism to delete the barred zero and double the hatted zeros.

$h(0) = 0$
$h(\hat{0}) = \hat{0}$
$h(0) = 0$
$h(1) = 1$

$g(0) = 0$
$g(\hat{0}) = 00$
$g(\hat{0}) = \epsilon$
$g(1) = 1$

$g(h^{-1}(L) \cap 0^i10^{2i}1) = \{0^i10^{2i}1\}$

(c) Convert $\{ww \mid w \in (a+b)^*\}$ into $\{a^nb^n \mid n \geq 1\}$

Place hats arbitrarily on $a$’s using an inverse homomorphism. Using intersection with a regular set, restrict $ww$ to the form $a^*b$ and force the second block of $a$’s to be hatted. Use a homomorphism to turn the hatted $a$’s into $b$’s and delete the original $b$’s.

$h(a) = a$
$h(\hat{a}) = a$
$h(b) = b$
\begin{align*}
g(a) &= a \\
g(\hat{a}) &= b \\
g(b) &= \epsilon \\
g(h^{-1}(L) \cap aa^*b\hat{a}^*a^*) &= \{a^n b^n \mid n \geq 1\}
\end{align*}

(d) For an arbitrary language \( L \subseteq (0 + 1)^* \), delete every other zero from each string of \( L \).

Assume this means to start with the 2nd zero and delete all even numbered zeros. Use an inverse homomorphism to hat zeros, intersection with a regular set to force hats on even numbered zeros, then another homomorphism to delete hatted zeros. Don’t forget when writing the regular expression that there may be no zeros, or an odd number of zeros.

\begin{align*}
h(0) &= 0 \\
h(\hat{0}) &= 0 \\
h(1) &= 1 \\
g(0) &= 0 \\
g(\hat{0}) &= \epsilon \\
g(1) &= 1 \\
g(h^{-1}(L) \cap 1^*(01^*\hat{0}1^*)^*(\epsilon + 0)1^*)
\end{align*}