Homework 6

5. Write a CFG for \{a^ib^jc^k \mid i \neq j \text{ or } i \neq k\}
   Use variables A, B, and C to generate zero or more a’s, b’s and c’s, respectively.

   \[
   \begin{align*}
   A & \rightarrow aA \mid \epsilon \\
   B & \rightarrow bB \mid \epsilon \\
   C & \rightarrow cC \mid \epsilon
   \end{align*}
   \]

   Break the language down into four cases:

   \[
   S \rightarrow WC \mid XC \mid Y \mid Z
   \]

   (i) Case $i > j$

   \[
   W \rightarrow aWb \mid aA
   \]

   (ii) Case $i < j$

   \[
   X \rightarrow aXb \mid bB
   \]

   (iii) Case $i > k$

   \[
   Y \rightarrow aYc \mid aAB
   \]

   (iv) Case $i < k$

   \[
   Z \rightarrow aZc \mid BCc
   \]

6. Write a CFG for the set of all strings of a’s, b’s and c’s not contained in \{wcw^R \mid w \in (a + b)^*\}.
   Generate the string from the outside in, creating pairs of characters. If all pairs are matched a’s or matched b’s, then we cannot produce a single c in the middle. Otherwise, we can generate an arbitrary string.

   \[
   \begin{align*}
   S & \rightarrow aSa \mid bSb \mid aEb \mid aEc \mid bEc \mid bEa \mid cEc \mid cEb \mid cEc \\
   S & \rightarrow a \mid b \mid \epsilon \\
   E & \rightarrow aE \mid bE \mid cE \mid \epsilon
   \end{align*}
   \]