Exercise 9.2.4

First observe that any proof that one of the languages is recursive generalizes to each $L_i$. So let’s prove $L_1$ to be recursive.

By the construction in Exercise 9.2.6 (a), we know the RE languages are closed under set union. This means the union of $L_2$, $L_3$, ..., $L_k$ yields an RE language. This resulting language is the complement of $L_1$. Theorem 9.4 states that if a language and its complement are both RE, then both languages are also recursive. Thus we conclude $L_1$ to be recursive.

There is an alternative solution found on the textbook’s website. Take TM’s $M_1$, $M_2$, ..., $M_K$ for each of the languages $L_1$, $L_2$, ..., $L_K$, respectively. Design a TM $M$ with $k$ tapes that accepts $L_1$ and always halts. $M$ copies its input to all the tapes and simulates $M_i$ on the $i$th tape. If $M_i$ accepts, then $M$ accepts. If any of the other TM’s accepts, $M$ halts without accepting. The problem statement (parts 1 and 2) assures that every string appears in exactly one of the languages so we know exactly one of the $M_i$ will accept. Therefore $M$ is sure to halt and we conclude that $L_1$ is recursive.