Solution for Hwk5 4.2.2

Solutions based on either DFAs or Homomorphisms were accepted.

Solution using DFAs:

Given a DFA $M$ such that $L(M) = L$, we will create a new DFA $N$ from $M$ such that $L(N) = L/a$.

Let $M = \{Q, \Sigma, q_0, \delta, F_M\}$, then $N = \{Q, \Sigma, q_0, \delta, F_N\}$ where $F_N = \{q \in Q | \delta(q, a) \in F_M\}$.

Note that this is not really a proof because we have not proven that $L(N) = L/a$. However, the minimal answer above was accepted.

Solution using a Homomorphism:

Define two homomorphisms $h_1$ and $h_2$:

$\forall \sigma \in \Sigma_L$, $h_1(\sigma) = \sigma$, $h_1(\hat{a}) = a$

$\forall \sigma \in \Sigma_L$, $h_2(\sigma) = \sigma$, $h_2(\hat{a}) = \varepsilon$

$h_1^{-1}(s)$ is the set of all permutations of ‘a’ with and without hats in $s$.

ex) let $\Sigma_L = \{a, b\}$, then $h_1^{-1}(aba) = \{aba, ab\hat{a}, \hat{a}ba, \hat{a}b\hat{a}\}$

Now we only want $\hat{a}$ at the end of the string, so we intersect $h_1^{-1}(L)$ with $\Sigma_L^*\hat{a}$.

ex) $h_1^{-1}(aba) \cap \Sigma_L^*\hat{a} = \{ab\hat{a}\}$

Finally, we apply $h_2$ to the intersection to remove $\hat{a}$.

Answer: $L/a = h_2( h_1^{-1}(L) \cap \Sigma_L^*\hat{a} )$

ex) $h_2( h_1^{-1}(aba) \cap \Sigma_L^*\hat{a} ) = \{ab\}$