Grader: Michael

4.2.13 part a

From: http://www-db.stanford.edu/ullman/ialcsols/sol4.html

Start out by complementing this language. The result is the language consisting of all strings of 0’s and 1’s that are not in \(0^*1^*\), plus the strings in \(L_{0n1n}\). If we intersect with \(0^*1^*\), the result is exactly \(L_{0n1n}\). Since complementation and intersection with a regular set preserve regularity, if the given language were regular then so would be \(L_{0n1n}\). Since we know the latter is false, we conclude the given language is not regular.

Comments: Some wrote it was enough to take the complement of the language. Note that 110 is in the complement of the language, so you have to do the intersection. Also, it is extremely important that people understand what a homomorphism in this context is. Think of a homomorphism as a function from some alphabet to another alphabet that is completely described by specifying what it does to the symbols in the domain alphabet because of the important relation \(h(ab)= h(a)h(b)\) for all symbols \(a,b\) of the domain alphabet. You cannot define a homomorphism by the number of symbols it is given in a particular instance, for example, and you should not define it with strings. Just specify what it does to each symbol in the alphabet it is defined on, then you can figure out what it does to strings using the relation. Note that it is okay for a homomorphism to send a symbol to a string, though.

4.2.13 part b

Let \(S = \{0^n1^m2^{n-m} \mid n \geq m \geq 0\}\)

Define the homomorphism \(h\): \(h(0) = 0, h(1) = 1, h(2) = 1\). Then 
\[h(S) = \{h(0^n1^m2^{n-m}) \mid n \geq m \geq 0\} = \{h(0^n)h(1^m)h(2^{n-m}) \mid n \geq m \geq 0\} = \{0^n1^m \mid n \geq m \geq 0\} = L_{0n1n}\.

Since the homomorphic image of a regular language is a regular language, if \(S\) were regular, \(L_{0n1n}\) would be regular, but it is not, thus \(S\) is not regular.
Comments: A few people just intersected the set with $L_{0n1n}$, and wrote that since the result is not regular, the original set is not regular. This is wrong because we already know $L_{0n1n}$ is not regular, and so the intersection may not be regular. So in other words, it is fruitless to intersect a regular set with a set that is not regular if you are giving this type of argument. Also, note that a regular set may contain a set than is not regular, so in most cases don’t try to use arguments involving subsets. Some people tried to break it up into cases: “if $m$ is not equal to $n$ then we have part a.” This is not true, since we must have $n > m$. 