1. Given a language $L \subseteq \Sigma^*$ let $L_{\text{init}} = \{w : \text{there exists some } x \in \Sigma^* \text{ so that } wx \in L\}$ (that is, the set of all initial segments of words in $L$).

(i) Prove that if two strings $x,y$ are $R_L$-equivalent then they are also $R_{L_{\text{init}}}$-equivalent.

(ii) Prove that if $L$ is regular then so is $L_{\text{init}}$.

2. For each of the following languages $L$, find a set of strings $S_L$ that contains exactly one string from every equivalence class of $R_L$:

(i) $L = (01+101)^*$

(ii) $L = \{a^n b^{2n} : n \in \mathbb{N}\}$

3. Recall that a language $L$ is called “boring” if for every $l \in \mathbb{N}$ there exists some $k \in \mathbb{N}$ such that all the strings whose lengths are between $k$ and $k+l$ belong to $L$.

Prove that if $L$ is a boring CFL then for some $n \in \mathbb{N}$ every string $w$ of length greater than $n$ belongs to $L$.

4. Construct a grammar $G$ such that $L(G) = \{0^n 1^{2n} : n \in \mathbb{N}\}$. Prove that this is indeed the language that your grammar generates.

5. Prove that $\{0^n 1^{2n} : n \in \mathbb{N}\}$ is not a CFL.