1. Describe an algorithm that on input -a DFA, M, outputs the answer to: "Is L(M) infinite?"

   (HINT: As a first step prove that L(M) is infinite if and only if \( L(M) \cap \{ w : n \leq |w| \leq 2n \} \neq \emptyset \) (where \( n \) is the number of states in M).

2. Construct Turing machines that compute the following languages:

   (a) \( \{ a^{2^n} : n \in \mathbb{N} \} \)

   (b) \( \{ a^n b^k c^{n+k} : n, k \in \mathbb{N} \} \)

   (c) \( \{ w \in \{0,1\}^* : |w| \text{ is even and there exists } i \leq \frac{|w|}{2} \text{ such that for all } j < i, a_j = a_{\frac{|w|}{2} + j} \text{ and } a_i = 1 \text{ and } a_{\frac{|w|}{2} + i} = 0 \} \) (where \( a_i \) is the \( i^{th} \) bit of \( w \)).