1. (i) Prove that the family of non-regular languages is closed under complementation (that is, if $L$ is non-regular then so is $L^c = \{ w : w \notin L \}$).

   (ii) Show that the family of non-regular languages is not closed under the union operation. That is, prove that there are non-regular $L_1, L_2$, such that $L_1 \cup L_2$ is regular.

   (iii) **BONUS:** Prove that there is a family $W$ of infinitely many non-regular languages such that $\cup \{ L : L \in W \}$ is regular.

2. Prove that if $L$ is a finite language then every DFA that computes $L$ must have at least $\max \{ |w| : w \in L \}$ many states.

3. For a word $w = o_1 \ldots o_n$, let $\overline{w}$ be the reverse word $\overline{w} = o_n \ldots o_1$. Prove that $\{ w\overline{w} : w \in \{0,1\}^* \}$ is not a regular language.

4. Prove that $L = \{ 0^k 1^n 0^n : k, n > 0 \} \cup \{ 1^i 0^j : i, j \geq 0 \}$ satisfies the requirements of the pumping lemma (that is, “there exists some $n \in N$ such that for every $w \in L$ if $|w| > n$ then there are $x, y, z$ such that:

   (i) $w = xyz$; (ii) $|xy| = n + 1$; (iii) For every $i \in N$, $xy^iz \in L$”). This language is not regular, but we will not prove it here.
5. Prove that \( \{ w : \#_0(w) - \#_1(w) \equiv 1 \mod 3 \} \) is regular but, on the other hand, \( \{ w : |\#_0(w) - \#_1(w)| \equiv 1 \mod 3 \} \) is not regular. **Hint:** consider \( 0^n1^{n+1} \) for sufficiently large \( n \).

6. Prove that for every regular expression \( r \) there exists a regular expression \( t \), such that: \( L(r) = \{ w : w \notin L(t) \} \).

7. Find a regular expression \( t \) over \( \{0,1\} \) such that: 
\[
L(t) = \{ w : w \notin L(((0+1)(0+1))^*) \}.
\]

8. For each of the following languages \( L \) (over \( \Sigma = \{o, p, q\} \)) find a regular expression \( r_L \) such that \( L(r_L) = L \):
   
   (i) \( L = \{ w : \text{if } p \text{ occurs in } w \text{ then } w \text{ ends with a } q \} \)
   
   (ii) \( L = \{ w : \#_p(w) \text{ is even} \} \)
   
   (iii) \( L = \{ w : \text{the next-to-last letter in } w \text{ is } p \} \)

9. For each of the following languages \( L \) describe the equivalence classes of \( R_L \) and determine the rank of \( R_L \):
   
   (i) \( L = \{ w \in \{0,1\}^* : w \text{ contains exactly two } 1\text{'s} \} \)
   
   (ii) \( L = \{ 0^m1^k0^{m+k} : m, k \in \mathbb{N} \} \)
   
   (iii) \( L = L(ab(a+b)^*ab) \)