1. Prove that if $L_1, L_2$ are regular languages, then so is:

   $L_1 \setminus L_2 = \{ w \in L_1 : w \notin L_2 \}$

2. Given a DFA, $M = (Q, \Sigma, q_0, \delta, F)$ and $p, q \in Q$, let $L(M, p, q) = \{ w : \hat{\delta}(p, w) = q \}$.

   Prove/refute each of the following claims:

   (i) For every $M, p, q$ as above and every $x, y \in \Sigma^*$, if $x \in L(M, p, q)$ and $y \in L(M, q, p)$ then $xy \in L(M, p, p)$

   (ii) For every $M, p, q$ as above and every $x, y, z \in \Sigma^*$, if $yz \in L(M, p, q)$ then there exist some $r \in Q$ such that for every $x \in L(M, r, r)$ and every $i \in \mathbb{N}$, $yx^iz \in L(M, p, q)$. 

3. Recall that a language is called “regular” if it is computable by some DFA.

   (i) Prove that any intersection of finitely many regular languages is a regular language.
(ii) Prove that there exist a set \( W \) of regular languages so that the
intersection of all languages in \( W \) is not regular.

(iii) **BONUS:** find a set \( W \) of regular languages such that \( W \) is infinite
and yet the intersection of all the languages in \( W \) is an infinite
regular language.

4. Find a set \( W \) consisting of infinitely many languages over \( \{0,1\} \) so that:
   (i) Each language in \( W \) is infinite
   (ii) Each language in \( W \) is regular (i.e. computable by some DFA)
   (iii) For every pair of languages \( L_1, L_2 \in W \), if \( L_1 \neq L_2 \) then \( L_1 \cap L_2 = \emptyset \)

5. Construct a DFA, \( M \), such that \( L(M) = L(N) \) where \( N \) is the following NFA:

![Diagram of NFA](image)

(Here \( \Sigma = \{a, b, c\} \))

6. Construct a NFA, \( M \), over \( \Sigma = \{1, 2, 3, 4, 5\} \) such that \( M \) has only 5 states
   and \( L(M) = \{w = \sigma_1 \sigma_2 \ldots \sigma_{|w|} : \text{for all } i < j < |w|, \ \sigma_i \leq \sigma_j \} \) (that is, the
   numbers that are the letters in \( w \) appear in increasing order).