

CS 381 – HW8 PART II  
(REVIEW PROBLEMS, NOT TO BE HANDED IN.)

1. For each of the following languages  $L$  over alphabet  $\{0, 1\}$ , give a DFA that accepts  $L$ .

- (a) The set of all strings that start and end with a zero.
- (b)  $L((0 + 1)^*(000)(0 + 1))$
- (c) The set of all strings such that every block of four consecutive symbols contains at least two 1's.

2.

- (i) Miscellaneous Exercise 5 in Kozen (p. 316)
- (ii) What is the language accepted by each of the NFA's in part (i)?
- (iii) Give an NFA that accepts the following language: the set of strings in  $(0 + 1)^*$  that contain a pair of 0's separated by a string of length divisible by 4.

3.

- (i) Describe in English the languages denoted by each of the following regular expressions.
- (ii) Construct an NFA or DFA for each of these languages.
  - (a)  $(11 + 0)^*(00 + 1)^*$
  - (b)  $(1 + 01 + 001)^*(\varepsilon + 0 + 00)$

4. Which of the following languages are regular. Prove your answer.

- (a)  $\{0^n \mid n \text{ is prime}\}$
- (b) the set of all strings which do not contain three consecutive 0's.
- (c)  $\{0^{3^n} \mid n \in \mathbb{N}\}$

5. Describe the equivalence classes under  $R_L$  for each of the languages  $L$  in problem 4. Prove.

6. Give a context-free grammar generating each of the following sets. Prove.

- (a) The set of palindromes (i.e., strings that read the same forward and backward) over alphabet  $\{a, b\}$
- (b)  $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$
- (c) The set of all strings over alphabet  $\{a, b\}$  with twice as many a's as b's.
- (d) The set of all strings over alphabet  $\{a, b\}$  not of the form  $ww$  for some string  $w$ .

7. For each language  $L$  in problem 6, give a PDA accepting  $L$ . Prove.

8. Which of the following are CFL's? Prove.

- (a)  $\{a^i b^j \mid j = i^2\}$

- (b)  $\{a^i \mid i \text{ is prime}\}$
- (c)  $\{a^i b^j \mid i \neq j \text{ and } i \neq 2j\}$

9. Design Turing machines to accept each of the following languages over alphabet  $\{0, 1\}$ .

- (a)  $\{0^n 1^n 0^n \mid n \geq 1\}$
- (b) The set of strings with an equal number of 0's and 1's.

9. Is it decidable for TM's  $M$  whether  $L(M) = \text{rev}(L(M))$ ? ( $\text{rev}(L) = \{\text{reverse}(w) \mid w \in L\}$ .) Prove.

These problems taken from *Introduction to Automata Theory, Languages, and Computation* by Hopcroft and Ullman and from the Kozen book.