Handout #9

Problem 1
(a)-(i)
(b)-(iii)
(c)-(v)
(d)-(iv)
(e)-(ii)

Problem 2
(ab*a + ba*b)* (ab* + ba*)

Handout #10

Problem 1
(a) \( h^{-1}(A) = \{\varepsilon\} \)
(b) \( h(B) = (01 + 0)^* \)
(c) \( h^{-1}(C) = a^* \)

Handout #11

Problem 1
The proof will be by contradiction. Assume that the languages are regular and are accepted by a DFA with \( n \) states. Let \( x \) be the sufficiently long string which will be used to give the contradiction.

(a) Let \( x = a^n b a^n \). Clearly \( x = \text{rev}(x) \). Hence we can apply pumping lemma to this string. Note that \( v \) will always consists of \( a \)'s to the left of \( b \). Hence pumping this region will result in a new string which has an asymmetrical distribution of \( a \)'s around \( b \). Hence the new string is not in the language. Hence the original language is not regular.

(b) Let \( x = (\varepsilon)^n \{ n \ “(” \ followed \ by \ n \ “)” \} \). In this case \( v \) will always consists of “(”. Hence the pumped up string has no longer equal number of right & left parenthesis, a necessary condition for balanced string of parentheses.

Problem 2
(1) No. Use homomorphism \( h(a) = 0 \) and \( h(b) = 11 \).
(2) No. Let \( L = \{a^n b^n | n \neq m \} \). Let \( L^C \) denote the complement of \( L \). Then if \( L \) is regular, it implies \( a^n b^n = a^* b^* \cap L^C \) is regular, a contradiction.
(3) No. Take \( z = a^n c a^n \) where \( n \) is the number of states in the DFA and show a contradiction using the pumping lemma.
(4) Yes.