Gates and Logic: From Transistors to Logic Gates and Logic Circuits

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Computer Science
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The slides are the product of many rounds of teaching CS 3410 by Professors Weatherspoon, Bala, Bracy, and Sirer.
Goals for Today

• From Switches to Logic Gates to Logic Circuits
• Logic Gates
  ▪ From switches
  ▪ Truth Tables
• Logic Circuits
  ▪ Identity Laws
  ▪ From Truth Tables to Circuits (Sum of Products)
• Logic Circuit Minimization
  ▪ Algebraic Manipulations
  ▪ Truth Tables (Karnaugh Maps)
• Transistors (electronic switch)
A switch

- Acts as a *conductor* or *insulator*
- Can be used to build amazing things...

The Bombe used to break the German Enigma machine during World War II
Basic Building Blocks: Switches to Logic Gates

- **Either (OR)**
  - Truth Table
    | A | B | Light |
    |---|---|-------|
    | OFF | OFF |      |
    | OFF | ON |      |
    | ON | OFF |      |
    | ON | ON |      |

- **Both (AND)**
  - Truth Table
    | A | B | Light |
    |---|---|-------|
    | OFF | OFF |      |
    | OFF | ON |      |
    | ON | OFF |      |
    | ON | ON |      |
Basic Building Blocks: Switches to Logic Gates

• Either (OR)

- Truth Table

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0 = OFF
1 = ON

• Both (AND)

- Truth Table

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Basic Building Blocks: Switches to Logic Gates

Did you know?

• George Boole Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.
Takeaway

• Binary (two symbols: **true** and **false**) is the basis of Logic Design
Building Functions: Logic Gates

- **NOT:**
  - Digital circuit that either allows a signal to pass through it or not.
  - Used to build logic functions
  - There are seven basic logic gates: AND, OR, NOT, NAND (not AND), NOR (not OR), XOR, and XNOR (not XOR) [later]

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- **AND:**

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- **OR:**

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- **NAND:**

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- **NOR:**

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Goals for Today

• From Switches to Logic Gates to Logic Circuits

• Logic Gates
  ▪ From switches
  ▪ Truth Tables

• Logic Circuits
  ▪ Identity Laws
  ▪ From Truth Tables to Circuits (Sum of Products)

• Logic Circuit Minimization
  ▪ Algebraic Manipulations
  ▪ Truth Tables (Karnaugh Maps)

• Transistors (electronic switch)
Next Goal

• Given a Logic function, create a Logic Circuit that implements the Logic Function...
• ...and, *with the minimum number of logic gates*
• Fewer gates: A cheaper ( $$$ ) circuit!
Logic Gates

NOT:

\[
\begin{array}{c|c}
A & \text{Out} \\
\hline
0 & 1 \\
1 & 0 \\
\end{array}
\]

AND:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]

NAND:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 1 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

OR:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

NOR:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]

XOR:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

XNOR:

\[
\begin{array}{c|c|c}
A & B & \text{Out} \\
\hline
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]
Logic Implementation
• How to implement a desired logic function?

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**Logic Implementation**

- How to implement a desired logic function?

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1) Write minterms
2) sum of products:
   - OR of all minterms where out=1
Logic Equations

• NOT:
  - out = \overline{a} = !a = \neg a

• AND:
  - out = a \cdot b = a \& b = a \land b

• OR:
  - out = a + b = a | b = a \lor b

• XOR:
  - out = a \oplus b = a\overline{b} + \overline{a}b

NAND:
  - out = \overline{a \cdot b} = !(a \& b) = \neg (a \land b)

NOR:
  - out = \overline{a + b} = !(a | b) = \neg (a \lor b)

XNOR:
  - out = a \oplus b = ab + \overline{ab}

• Logic Equations
  - Constants: true = 1, false = 0
  - Variables: a, b, out, ...
  - Operators (above): AND, OR, NOT, etc.
Identities
Identities useful for manipulating logic equations
  – For optimization & ease of implementation

\( a + 0 = \)
\( a + 1 = \)
\( a + \bar{a} = \)

\( a \cdot 0 = \)
\( a \cdot 1 = \)
\( a \cdot \bar{a} = \)
Identities
Identities useful for manipulating logic equations
  – For optimization & ease of implementation

\[(a + b) = (a \cdot b) = a + a \cdot b = a(b+c) = a(b + c)\]
Goals for Today

• From Switches to Logic Gates to Logic Circuits

• Logic Gates
  ▪ From switches
  ▪ Truth Tables

• Logic Circuits
  ▪ From Truth Tables to Circuits (Sum of Products)
  ▪ Identity Laws

• Logic Circuit Minimization – why?
  ▪ Algebraic Manipulations
  ▪ Truth Tables (Karnaugh Maps)

• Transistors (electronic switch)
## Checking Equality w/Truth Tables

**circuits ↔ truth tables ↔ equations**

**Example:** \((a+b)(a+c) = a + bc\)

<table>
<thead>
<tr>
<th>(a)</th>
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Takeaway

• Binary (two symbols: \textcolor{orange}{true} and \textcolor{orange}{false}) is the basis of Logic Design

• More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.
Goals for Today

• From Switches to Logic Gates to Logic Circuits
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  ▪ Truth Tables (Karnaugh Maps)
• Transistors (electronic switch)
Next Goal

• How to standardize minimizing logic circuits?
Karnaugh Maps

How does one find the most efficient equation?
– Manipulate algebraically until...?
– Use Karnaugh Maps (optimize visually)
– Use a software optimizer

For large circuits
– Decomposition & reuse of building blocks
Minimization with Karnaugh maps (1)

Sum of minterms yields

\[ \text{out} = \]

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Minimization with Karnaugh maps (2)

Sum of minterms yields
- \[ \text{out} = \overline{abc} + \overline{abc} + \overline{abc} + \overline{abc} \]

Karnaugh map minimization
- Cover all 1’s
- Group adjacent blocks of \(2^n\) 1’s that yield a rectangular shape
- Encode the common features of the rectangle
  - \[ \text{out} = a\bar{b} + \bar{ac} \]
Karnaugh Minimization Tricks (1)

- Minterms can overlap
  - **out** =

- Minterms can span 2, 4, 8 or more cells
  - **out** =

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Karnaugh Minimization Tricks (2)

- The map wraps around
  - out =

  - out =
Karnaugh Minimization Tricks (3)

- “Don’t care” values can be interpreted individually in whatever way is convenient
  - assume all x’s = 1
  - out =
  - assume middle x’s = 0
  - assume 4th column x = 1
  - out =
Minimization with K-Maps

Rules:
• Use fewest circles necessary to cover all 1’s
• Circles must cover only 1’s
• Circles span rectangles of size power of 2 (1, 2, 4, 8…)
• Circles should be as large as possible (all circles of 1?)
• Circles may wrap around edges of K-Map
• 1 may be circled multiple times if that means fewer circles

(1) Circle the 1’s (see below)
(2) Each circle is a logical component of the final equation

\[ = \bar{a}b + \bar{a}c \]
Multiplexer

- A multiplexer selects between multiple inputs
  - out = a, if d = 0
  - out = b, if d = 1

- Build truth table
- Minimize diagram
- Derive logic diagram
Takeaway

• Binary (two symbols: true and false) is the basis of Logic Design.

• More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.

• Any logic function can be implemented as “sum of products”. Karnaugh Maps minimize number of gates.
Goals for Today

• From Transistors to Gates to Logic Circuits
• Logic Gates
  ▪ From transistors
  ▪ Truth Tables
• Logic Circuits
  ▪ From Truth Tables to Circuits (Sum of Products)
  ▪ Identity Laws
• Logic Circuit Minimization
  ▪ Algebraic Manipulations
  ▪ Truth Tables (Karnaugh Maps)
• Transistors (electronic switch)
Silicon Valley & the Semiconductor Industry

• Transistors:
  • Youtube video “How does a transistor work”
    https://www.youtube.com/watch?v=IcrBqCFLHIY
  • Break: show some Transistor, Fab, Wafer photos
Transistors 101

**N-Type Silicon:** negative free-carriers (electrons)

**P-Type Silicon:** positive free-carriers (holes)

**P-Transistor:** negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain

**N-Transistor:** works the opposite way

Metal-Oxide Semiconductor (Gate-Insulator-Silicon)

- Complementary MOS = **CMOS** technology uses both p- & n-type transistors
CMOS Notation

N-type

\[
\text{gate} \quad \text{Off/Open} \quad 0 \quad \text{On/Closed} \quad 1
\]

P-type

\[
\text{gate} \quad \text{Off/Open} \quad 1 \quad \text{On/Closed} \quad 0
\]

Gate input controls whether current can flow between the other two terminals or not.

*Hint:* the “o” bubble of the p-type tells you that this gate wants a 0 to be turned on
2-Transistor Combination: NOT

- Logic gates are constructed by combining transistors in complementary arrangements
- Combine p&n transistors to make a NOT gate:

**CMOS Inverter**

- **power source (1)**
- **p-gate**
- **n-gate**
- **ground (0)**

- **input**
- **output**

Diagram:

1. **p-gate closes**: p-gate stays open
2. **n-gate stays open**: n-gate closes
3. **ground (0)**

Input: 0 → Output: 1
Input: 1 → Output: 0

Diagram:

1. **p-gate stays open**: n-gate closes
2. **ground (0)**

Input: 0 → Output: 1
Input: 1 → Output: 0
Inverter

Function: NOT

Symbol:

Truth Table:

<table>
<thead>
<tr>
<th>In</th>
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NOR Gate

Function: NOR
Symbol:

Truth Table:

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Building Functions (Revisited)

• **NOT:** 

• **AND:** 

• **OR:** 

• **NAND and NOR are universal**
  - Can implement *any* function with NAND or just NOR gates
  - useful for manufacturing
Logic Gates

- One can buy gates separately
  - ex. 74xxx series of integrated circuits
  - cost ~$1 per chip, mostly for packaging and testing

- Cumbersome, but possible to build devices using gates put together manually
Then and Now

- **The first transistor**
  - One workbench at AT&T Bell Labs
  - 1947
  - Bardeen, Brattain, and Shockley

- **Intel Haswell**
  - 1.4 billion transistors, 22nm
  - 177 square millimeters
  - Four processing cores

https://en.wikipedia.org/wiki/Transistor_count
Then and Now

- The first transistor
  - One workbench at AT&T Bell Labs
  - 1947
  - Bardeen, Brattain, and Shockley

- Intel Broadwell
  - 7.2 billion transistors, 14nm
  - 456 square millimeters
  - Up to 22 processing cores


https://en.wikipedia.org/wiki/Transistor_count
Big Picture: Abstraction

• Hide complexity through simple abstractions
  
  ▪ Simplicity
    • Box diagram represents inputs and outputs
  
  ▪ Complexity
    • Hides underlying NMOS- and PMOS-transistors and atomic interactions
Summary

- Most modern devices made of billions of transistors
  - You will build a processor in this course!
  - Modern transistors made from semiconductor materials
  - Transistors used to make logic gates and logic circuits
- We can now implement any logic circuit
  - Use P- & N-transistors to implement NAND/NOR gates
  - Use NAND or NOR gates to implement the logic circuit
  - Efficiently: use K-maps to find required minimal terms