

# State and Finite State Machines

**Anne Bracy**

**CS 3410**

Computer Science

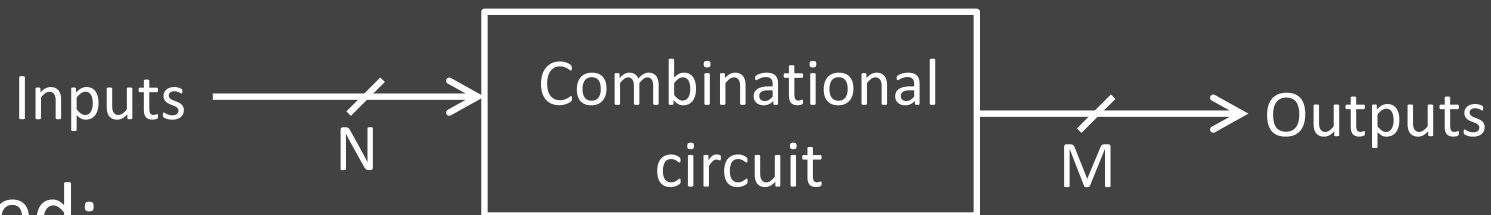
Cornell University

The slides are the product of many rounds of teaching CS 3410 by Professors Weatherspoon, Bala, Bracy, and Sirer.

# Stateful Components

## Combinational logic

- Output computed directly from inputs
- System has no internal state
- Nothing depends on the past!



Need:

- to record data
- to build **stateful** circuits
- a state-holding device

Enter: **Sequential Logic & Finite State Machines**

# Goals for Today

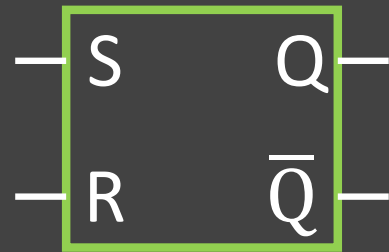
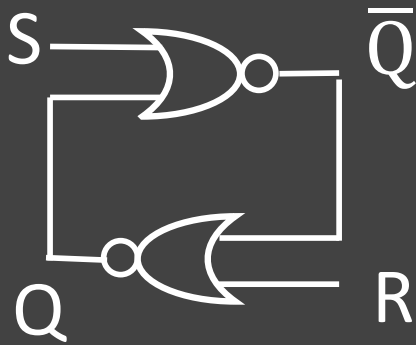
## State

- Storing 1 bit
  - Set-Reset Latch
  - D Latch
  - D Flip-Flops
- Storing N bits:
  - Registers
  - Memory

## Finite State Machines (FSM)

- Mealy (and Moore) Machines
- Serial Adder Example

# Round 1: Set-Reset (SR) Latch



Stores a value  $Q$  and its complement

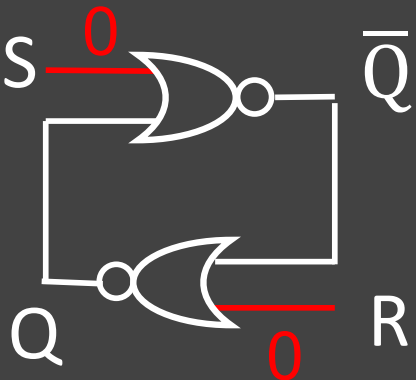
$S$	$R$	$Q$	$\bar{Q}$
0	0		
0	1		
1	0		
1	1		

For reference:

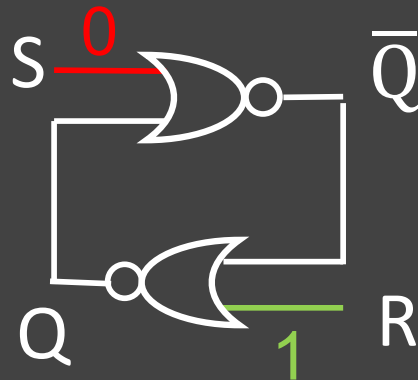
A	B	OR	NOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

If  $Q$  is 1, stays 1  
if  $Q$  is 0, stays 0

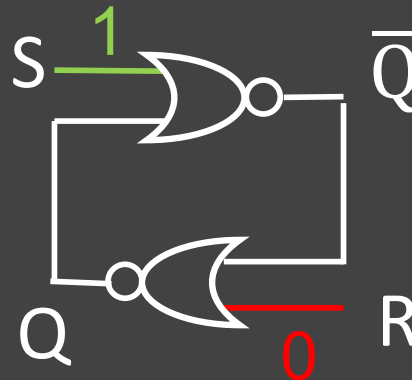
What happens when  $S, R$  changes from 1,1 to 0,0?  
 $Q, \bar{Q}$  become unstable, oscillate ( $0,0 \rightarrow 1,1 \rightarrow 0,0$ )



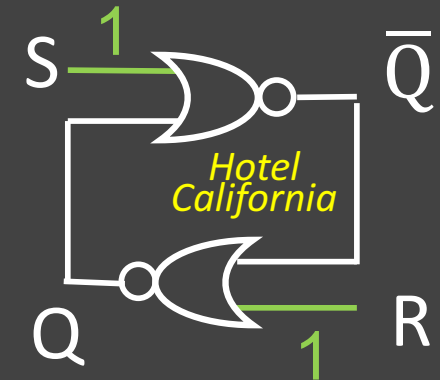
no change



Resets  $Q$



Sets  $Q$



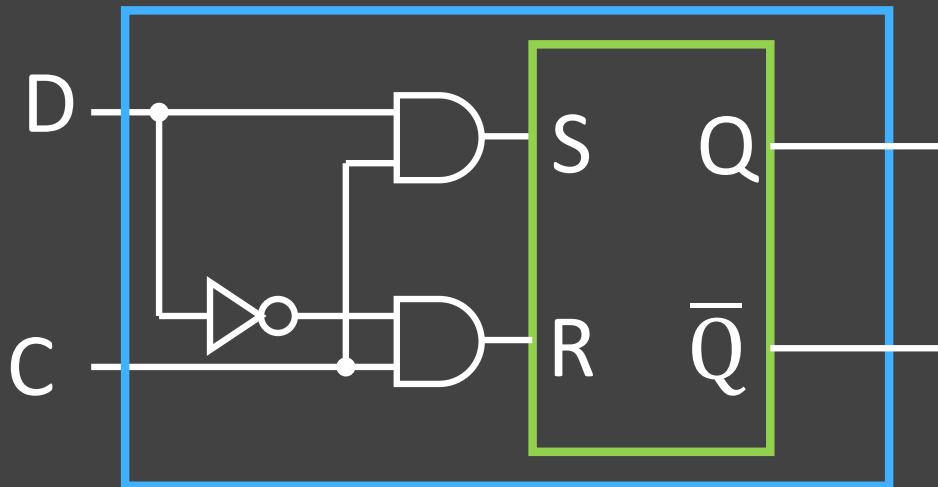
Forbidden!

# iClicker Question

What's wrong with the SR Latch?

- A. Q is undefined when  $S=1$  and  $R=1$   
(That's why this is called the forbidden state.)
- B. Q oscillates between 0 and 1 when the inputs transition from  $S=1$  and  $R=1 \rightarrow S=0$  and  $R=0$
- C. The SR Latch is problematic b/c it has two outputs to store a single bit.
- D. There is nothing wrong with the SR Latch!

# Round 2: D Latch (1)

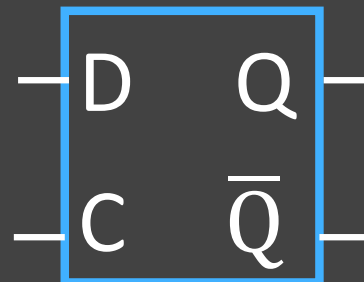


- Inverter prevents SR Latch from entering 1,1 state
- C = enables change

C	D	Q	$\bar{Q}$
0	0		
0	1		
1	0		
1	1		

C = 1, D Latch *transparent*:  
set/reset (according to D)

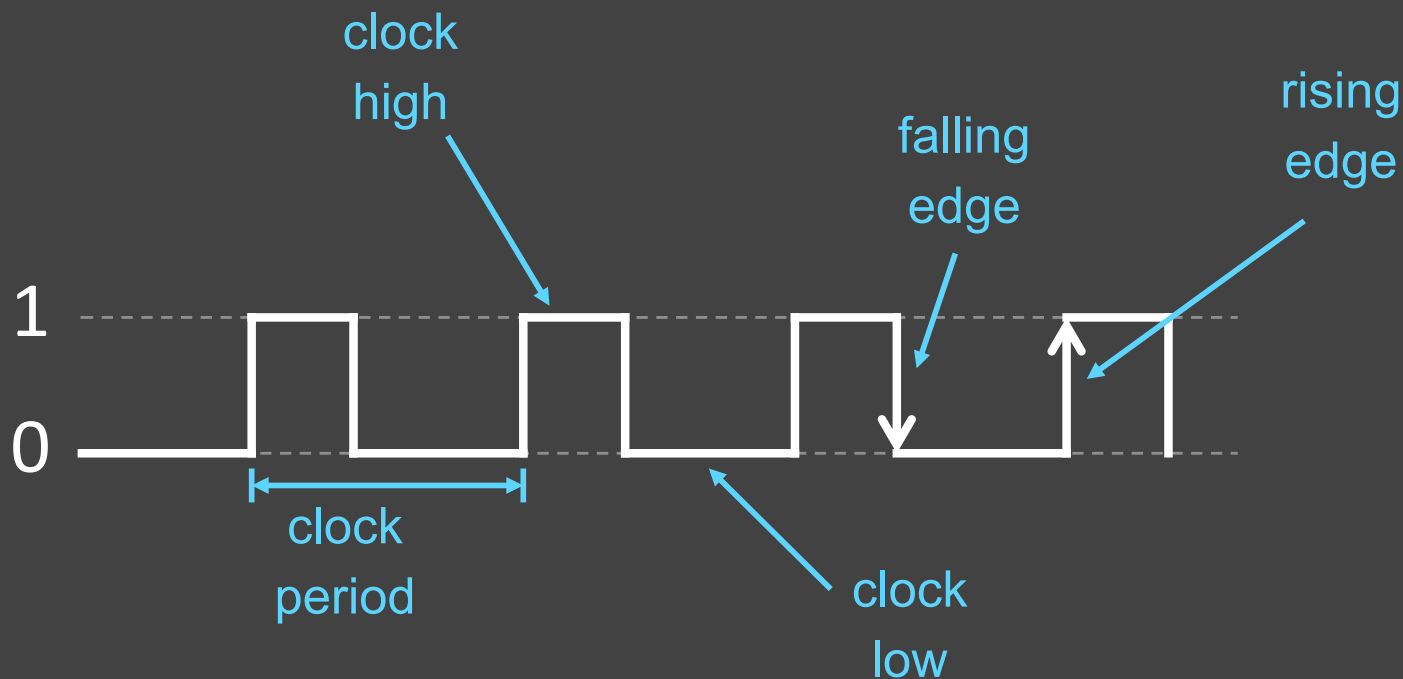
C = 0, D Latch *opaque*:  
keep state (ignore D)



# Aside: Clocks

Clock helps coordinate state changes

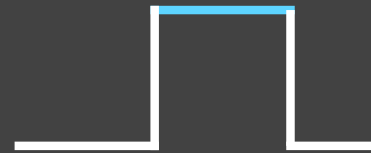
- Fixed period
- Frequency =  $1/\text{period}$



# Clock Disciplines

## Level sensitive

- State changes when clock is high (or low)



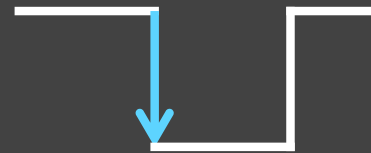
## Edge triggered

- State changes at clock edge

positive edge-triggered



negative edge-triggered

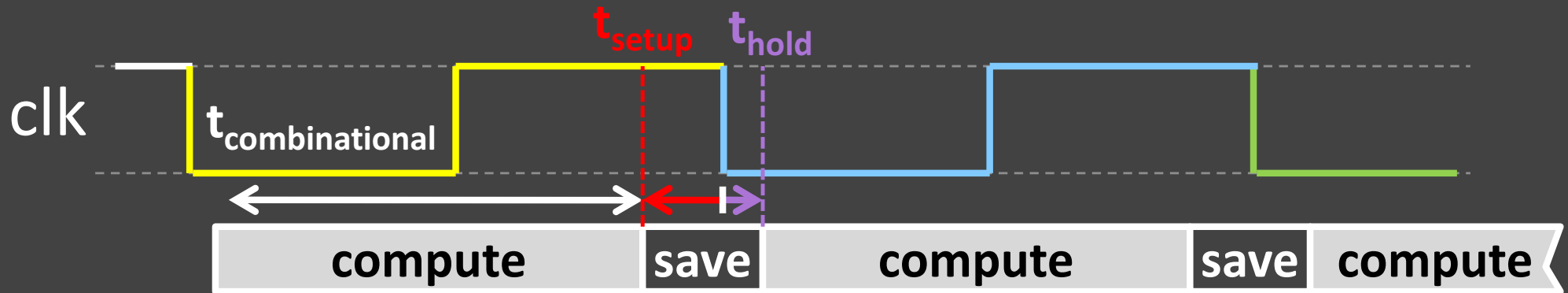




# Clock Methodology

## Clock Methodology

- Negative edge, synchronous

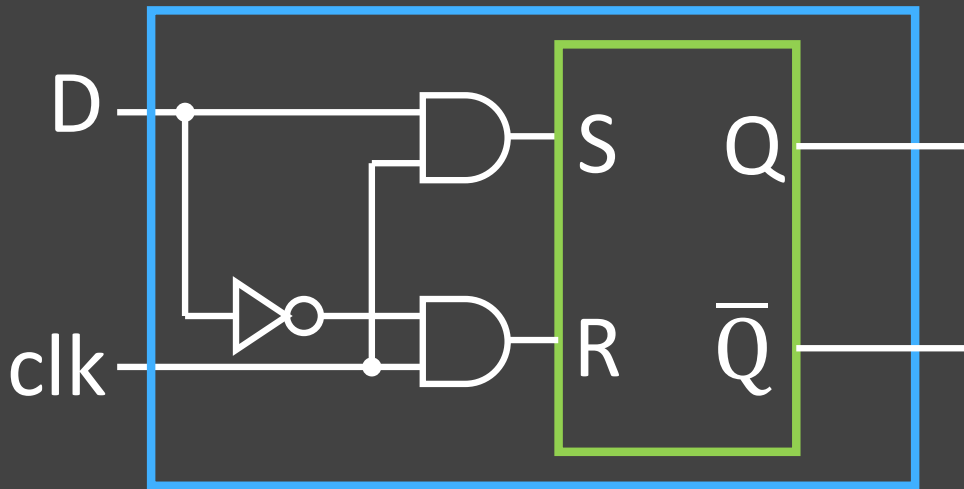


Edge-Triggered → signals must be stable near falling edge

“near” = **before** and **after**

$t_{\text{setup}}$   $t_{\text{hold}}$

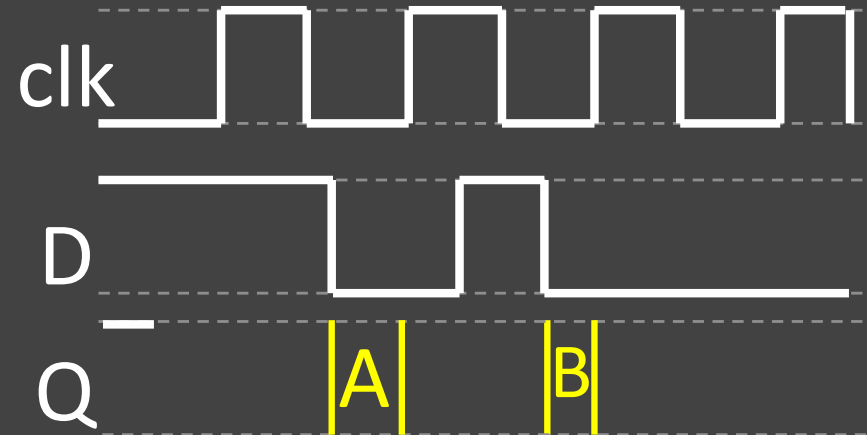
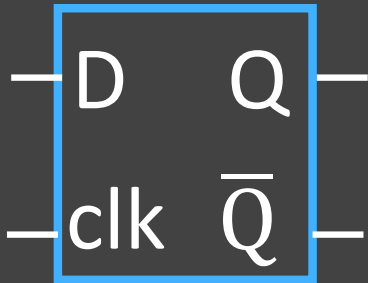
## Round 2: D Latch (2)



- Level sensitive
- Inverter prevents SR Latch from entering 1,1 state
- clk = enables change

clk	D	Q	$\bar{Q}$
0	0		
0	1		
1	0		
1	1		

# iClicker Question

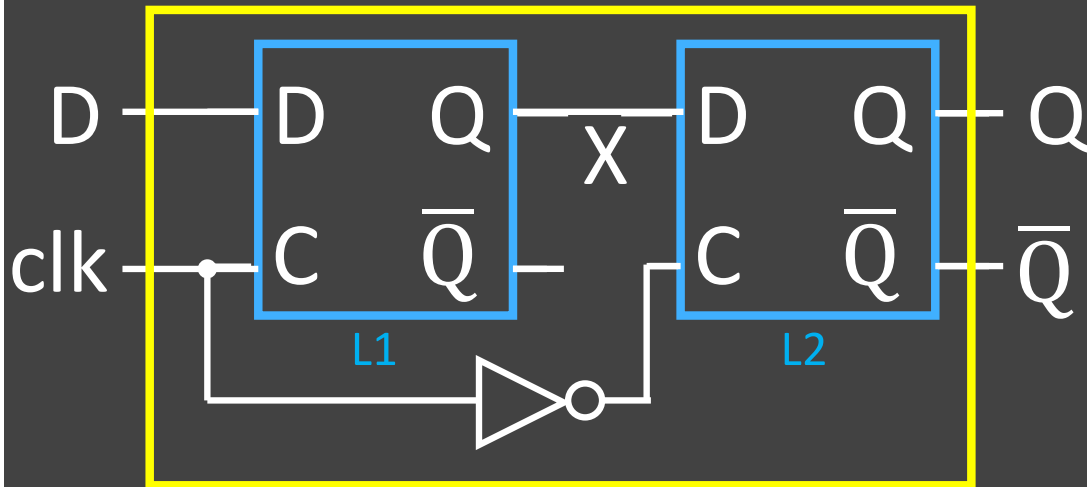


clk	D	Q	$\bar{Q}$
0	0	Q	$\bar{Q}$
0	1	Q	$\bar{Q}$
1	0	0	1
1	1	1	0

What is the value of Q at A & B?

- a)  $A = 0, B = 0$
- b)  $A = 0, B = 1$
- c)  $A = 1, B = 0$
- d)  $A = 1, B = 1$

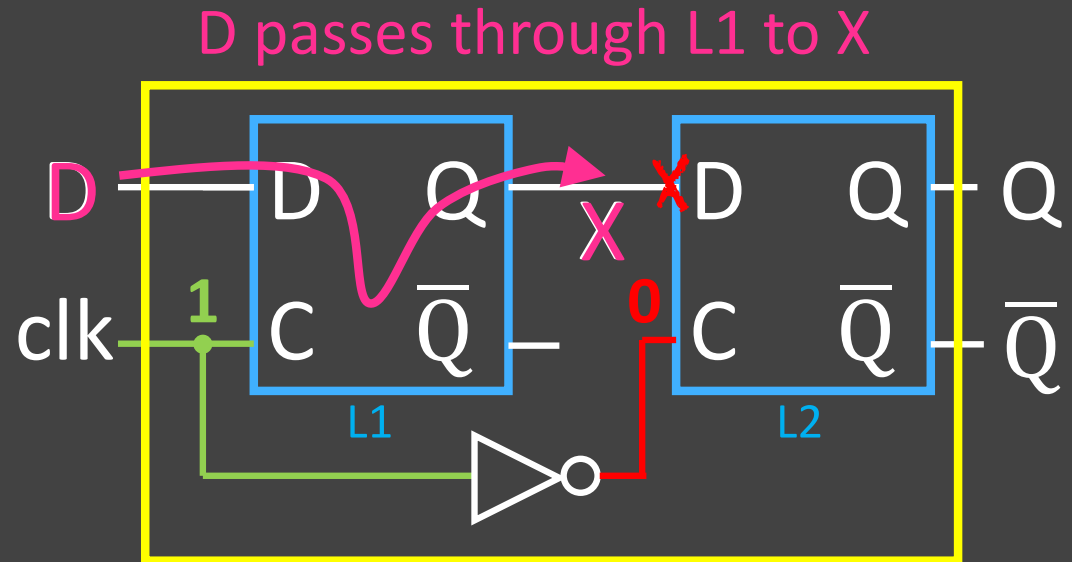
## Round 3: D Flip-Flop



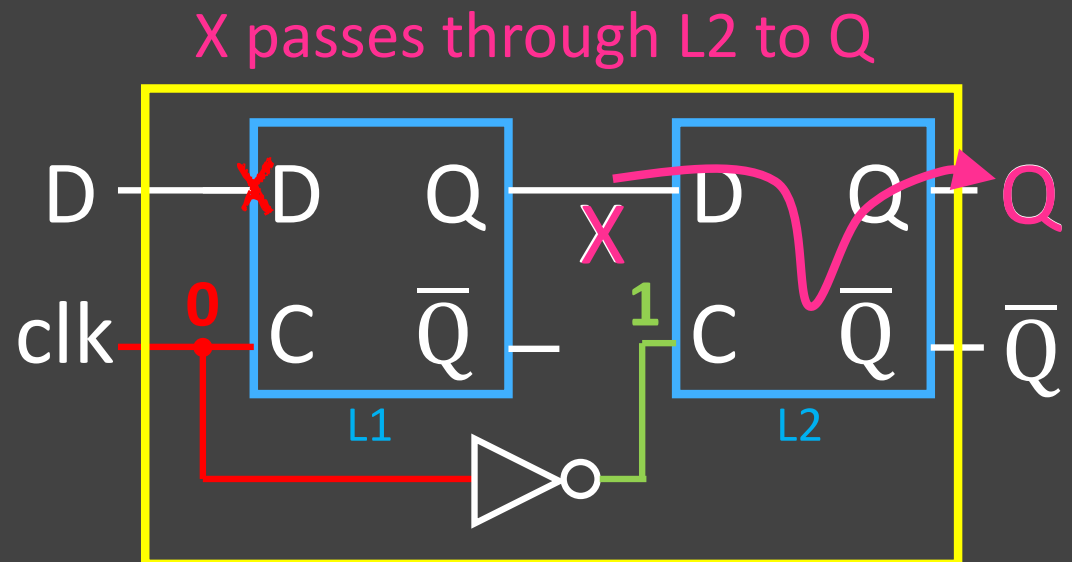
- Edge-Triggered
- Data captured when clock high
- Output changes only on falling edges

# Round 3: D Flip-Flop

Clock = 1: L1 *transparent*  
L2 *opaque*



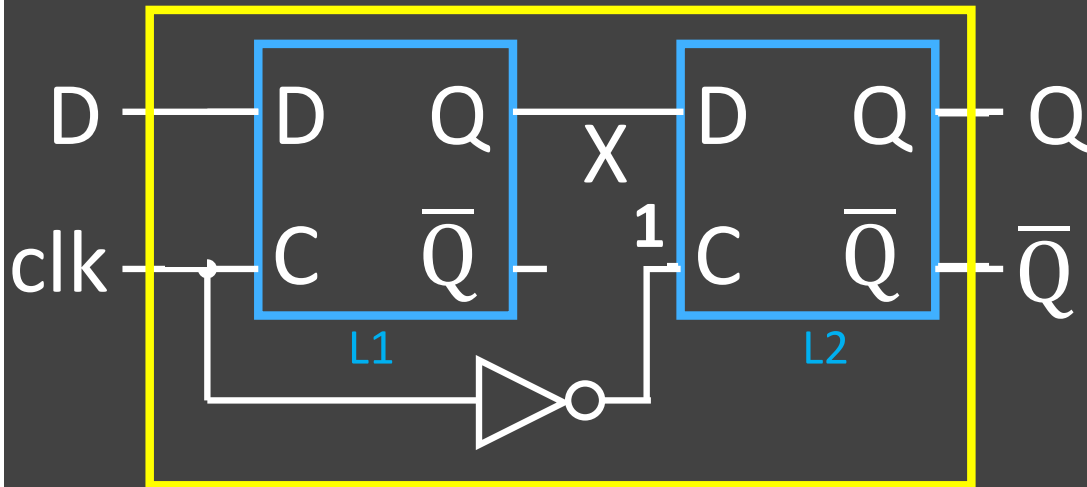
Clock = 0: L1 *opaque*  
L2 *transparent*



Thus, on edge of the clock  
(when **CLK** falls from 1→0)

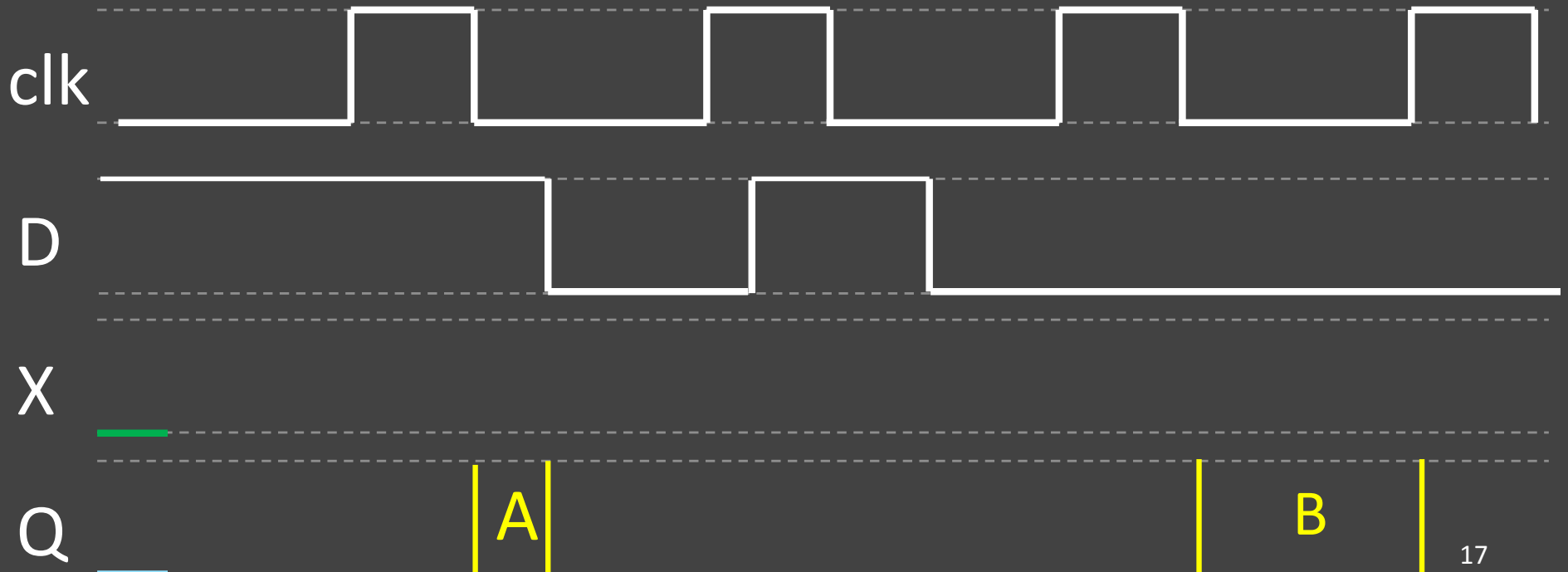
(D passes through to Q)

# iClicker Question



What is the value of Q at A & B?

- a)  $A = 0, B = 0$
- b)  $A = 0, B = 1$
- c)  $A = 1, B = 0$
- d)  $A = 1, B = 1$



# Goals for Today

## State

- Storing 1 bit
  - Set-Reset Latch
  - D Latch
  - D Flip-Flops
- Storing N bits:
  - Registers
  - Memory

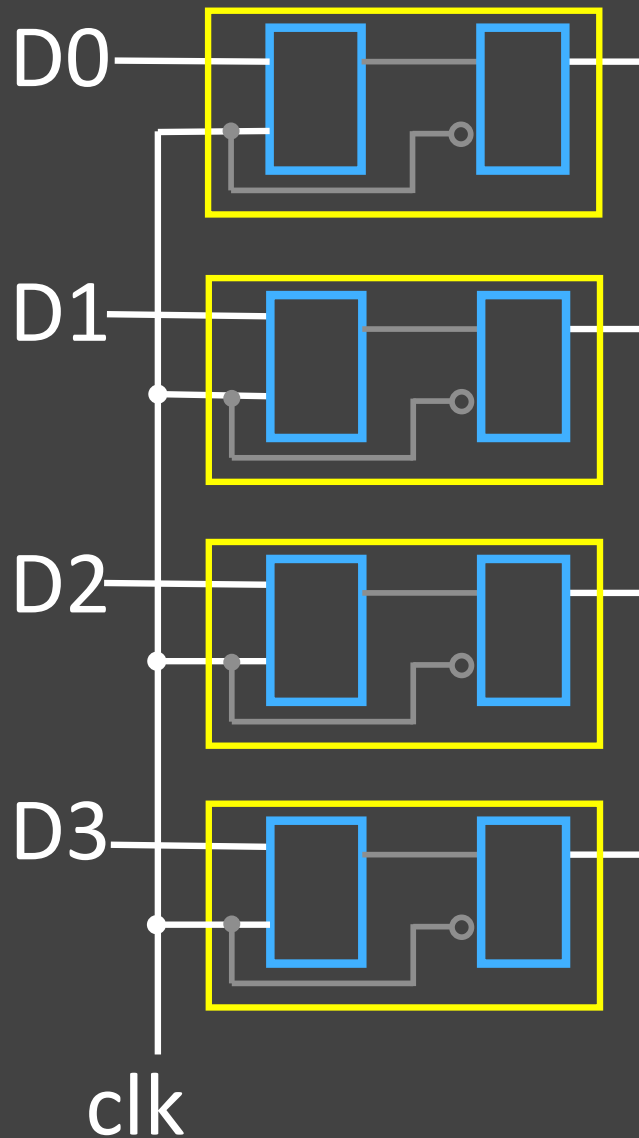
~~A word about clocks~~

A word about terminology

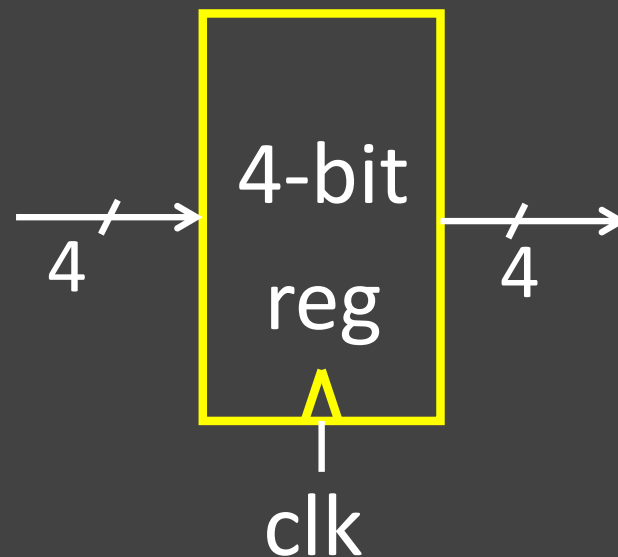
## Finite State Machines (FSM)

- Mealy and Moore Machines
- Serial Adder Example

# Registers



- D flip-flops in parallel
- shared clock
- Additional (optional) inputs: writeEnable, reset, ...

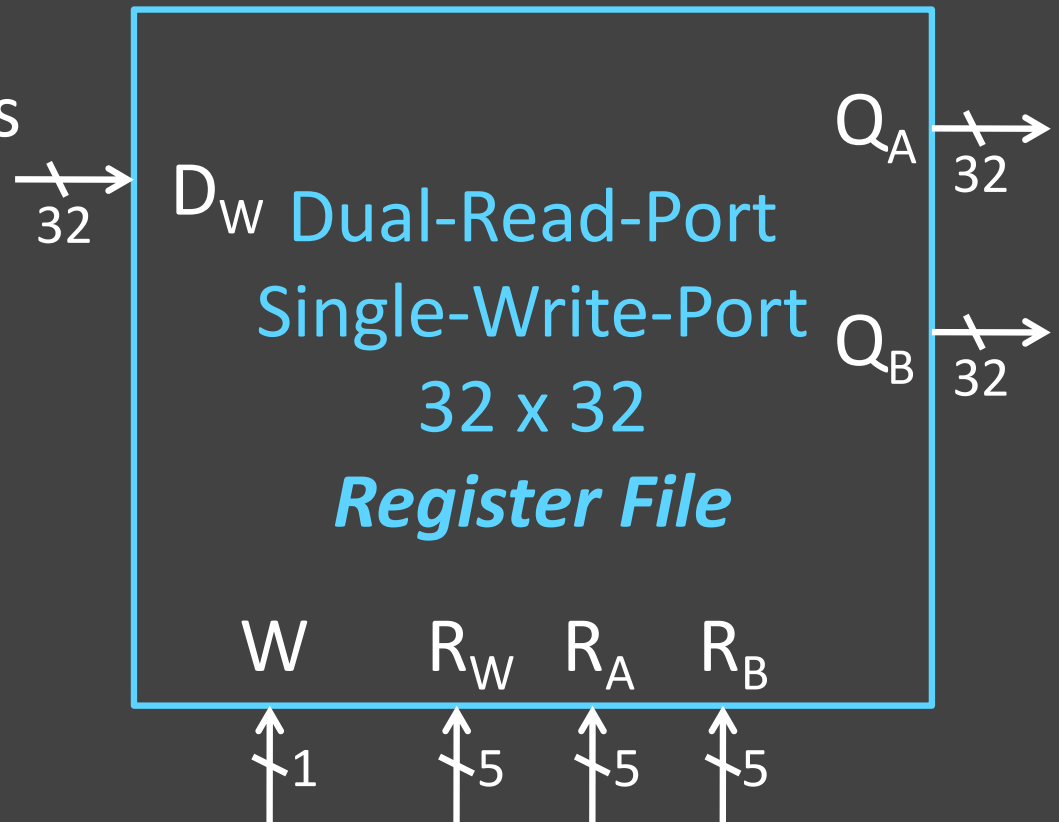




# Register File

## Register File

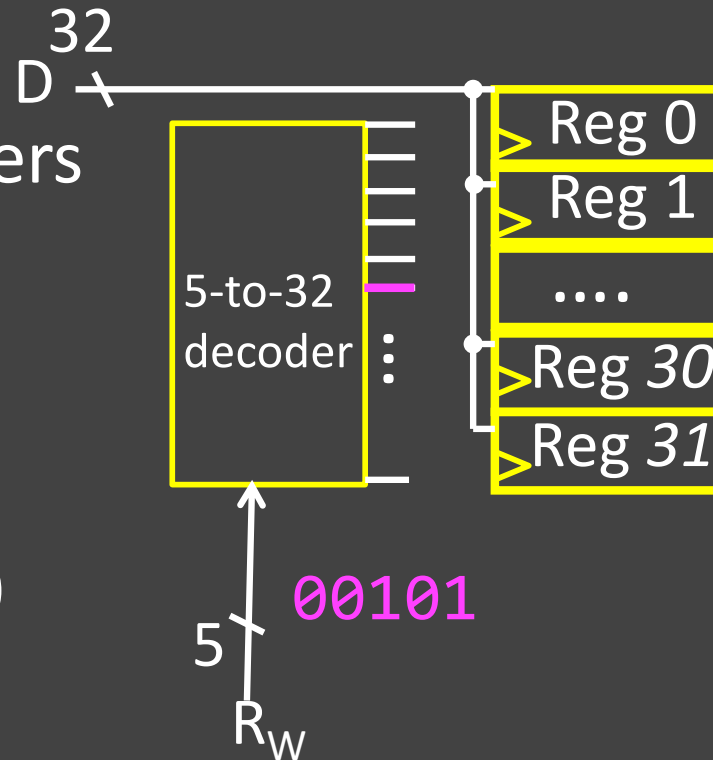
- N read/write registers
- Indexed by register number



# Writing to the Register File (1)

## Register File

- N read/write registers
- Indexed by register number



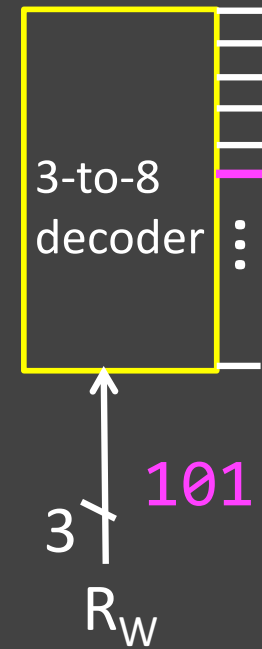
addi **r5**, r0, 10

How to write to **one** register in the register file?

- Need a decoder

# Activity: 3-to-8 decoder truth table & circuit

i2	i1	i0	o0	o1	o2	o3	o4	o5	o6	o7
0	0	0								
0	0	1								
0	1	0								
0	1	1								
1	0	0								
1	0	1								
1	1	0								
1	1	1								

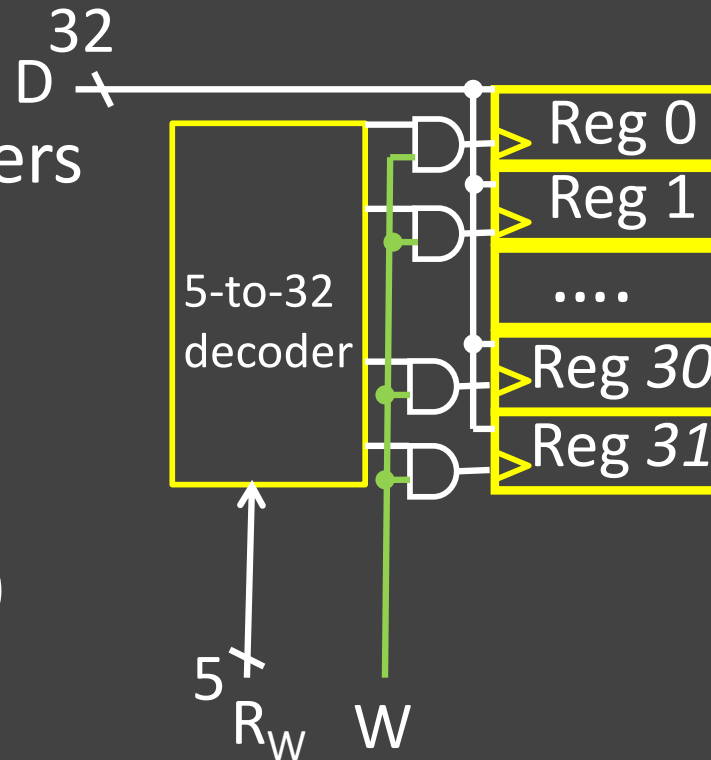


# Writing to the Register File (2)

## Register File

- N read/write registers
- Indexed by register number

addi **r5**, r0, 10



How to write to **one** register in the register file?

- Need a decoder

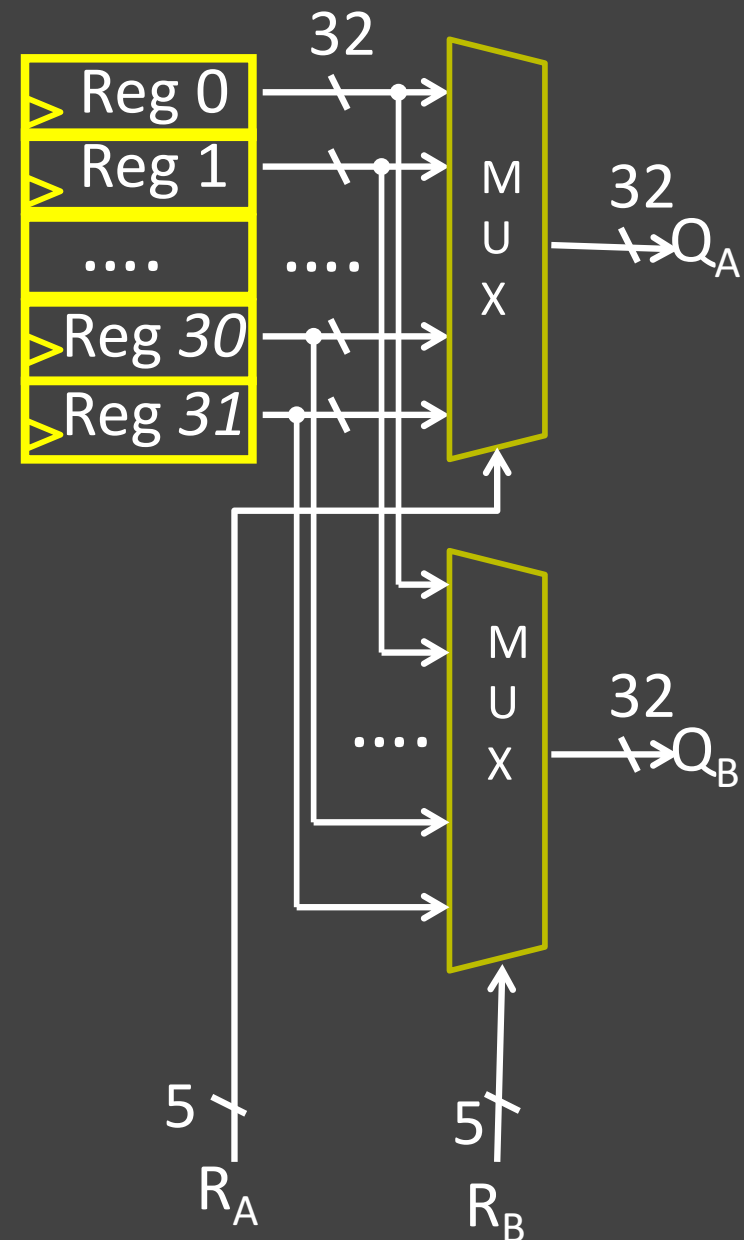
# Reading from the Register File

## Register File

- N read/write registers
- Indexed by register number

## How to read from two registers?

- Need a multiplexor



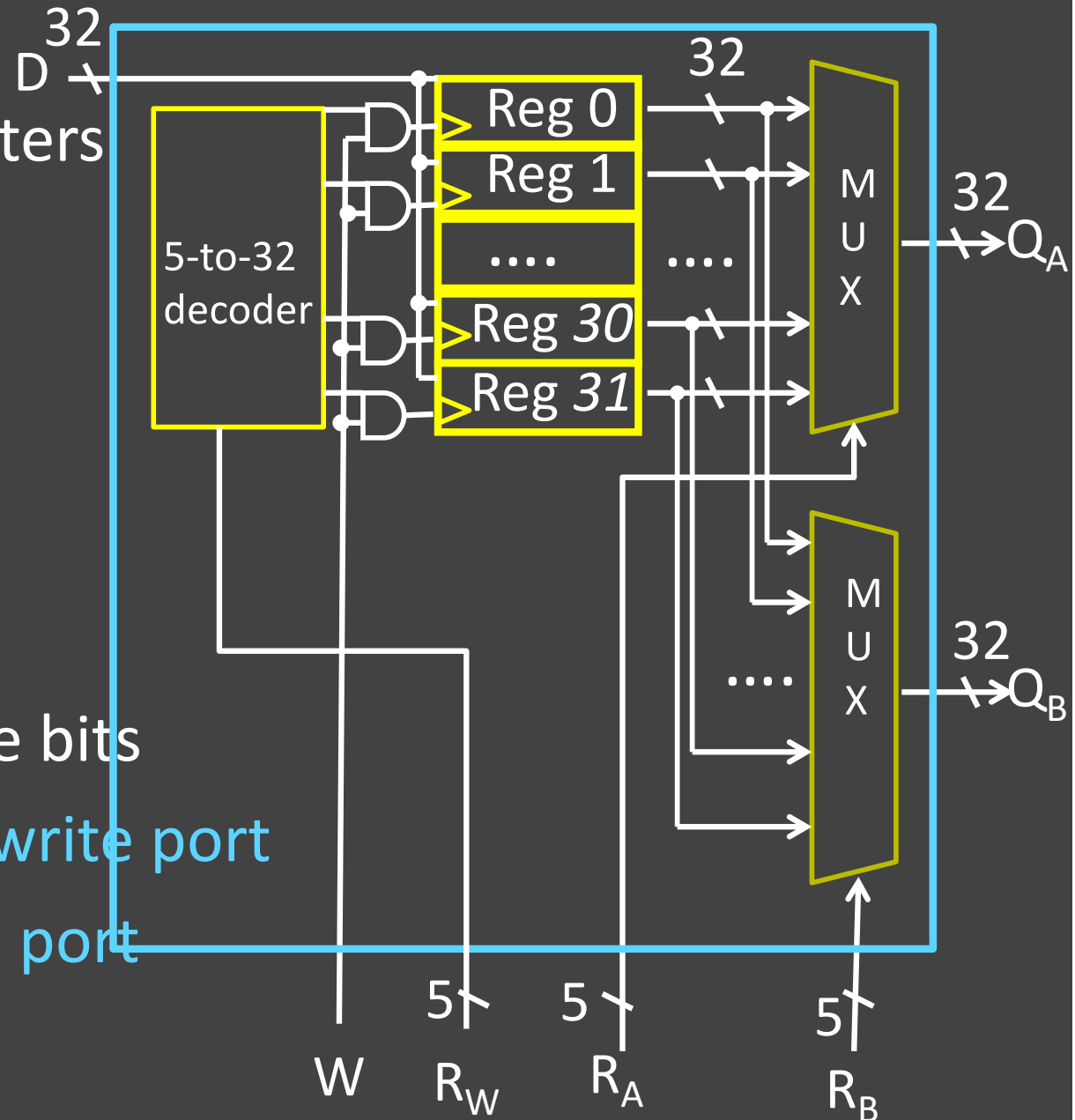
# Register File

## Register File

- N read/write registers
- Indexed by register number

## Implementation:

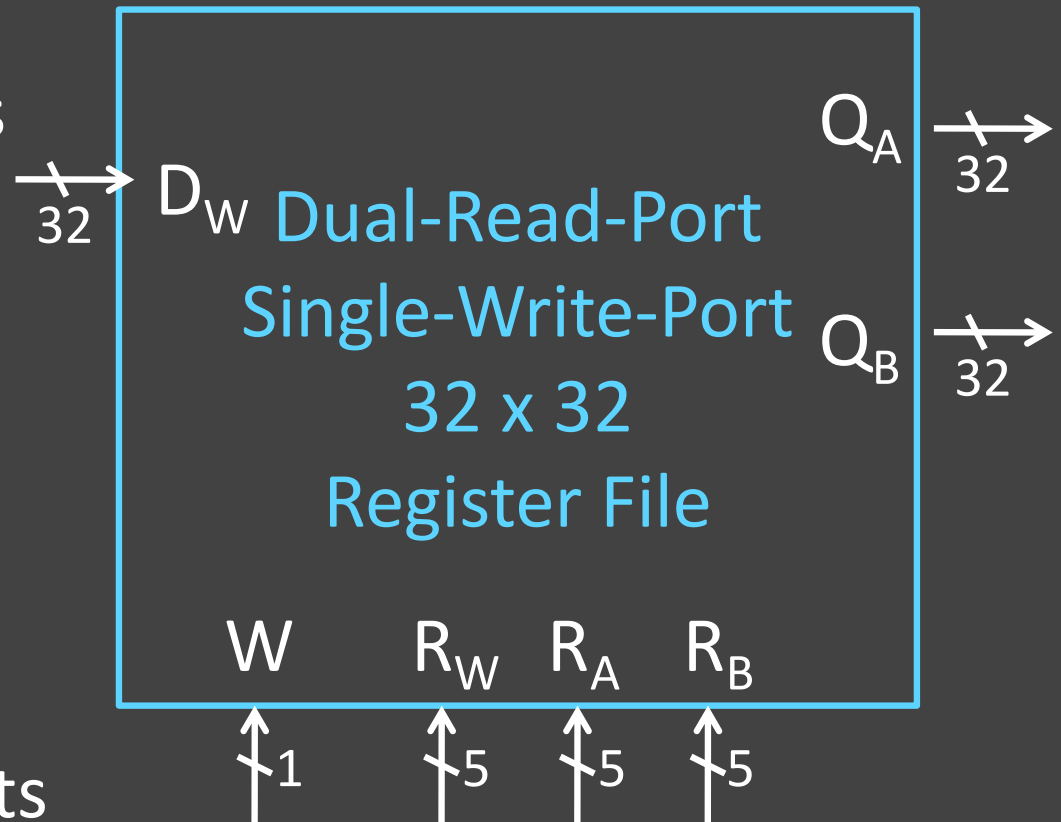
- D flip flops to store bits
- Decoder for each **write port**
- Mux for each **read port**



# Register File

## Register File

- N read/write registers
- Indexed by register number



## Implementation:

- D flip flops to store bits
- Decoder for each **write port**
- Mux for each **read port**

# Tradeoffs

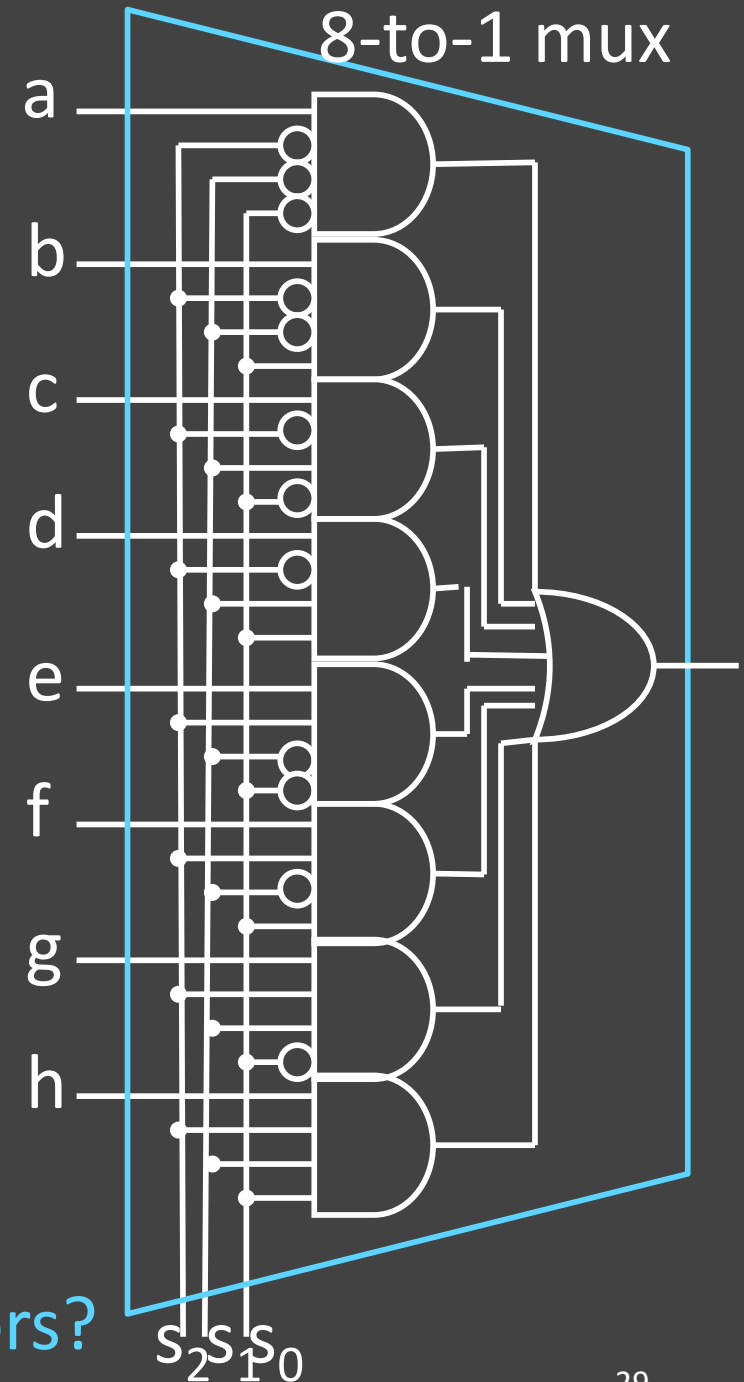
## Register File tradeoffs

- + Very fast (a few gate delays for both read and write)
- + Adding extra ports is straightforward
- Doesn't scale

e.g. 32Mb register file with  
32 bit registers

Need 32x 1M-to-1 multiplexor  
and 32x 20-to-1M decoder

How many logic gates/transistors?



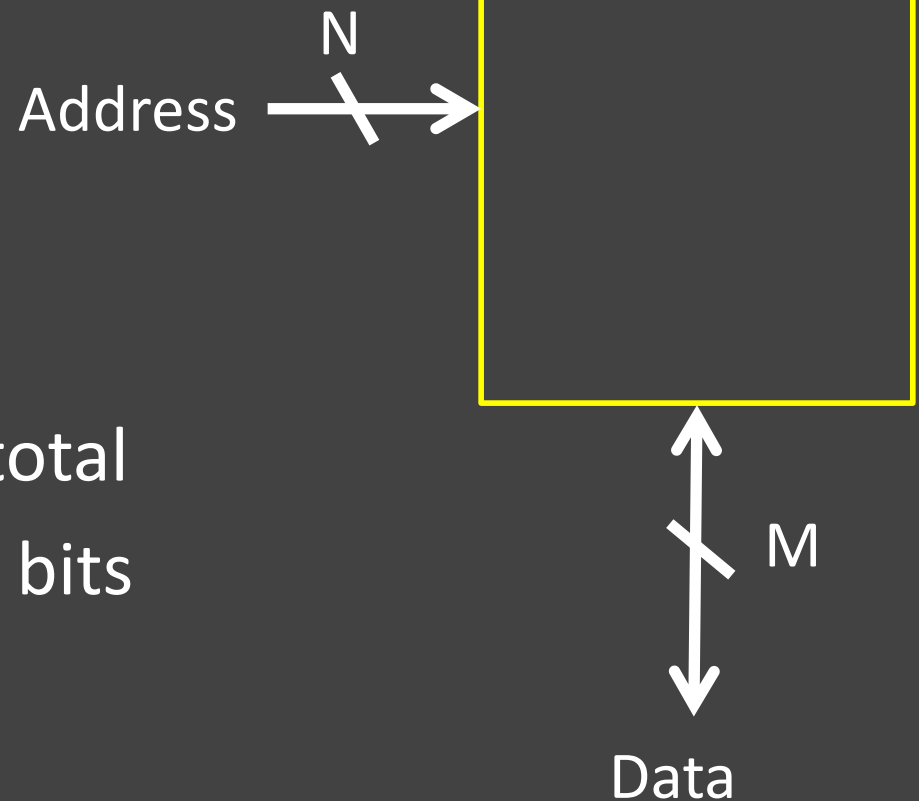


# Takeaways

- Set-Reset (SR) Latch can store one bit and we can change the value of the stored bit. But, SR Latch has a forbidden state.
- D Latch can store and change a bit like an SR Latch while avoiding a forbidden state.
- A D Flip-Flop stores one bit. The bit can be changed in a synchronized fashion on the edge of a clock signal.
- An  $N$ -bit **register** stores  $N$ -bits. It is created with  $N$  D-Flip-Flops in parallel plus a shared clock.

# Memory

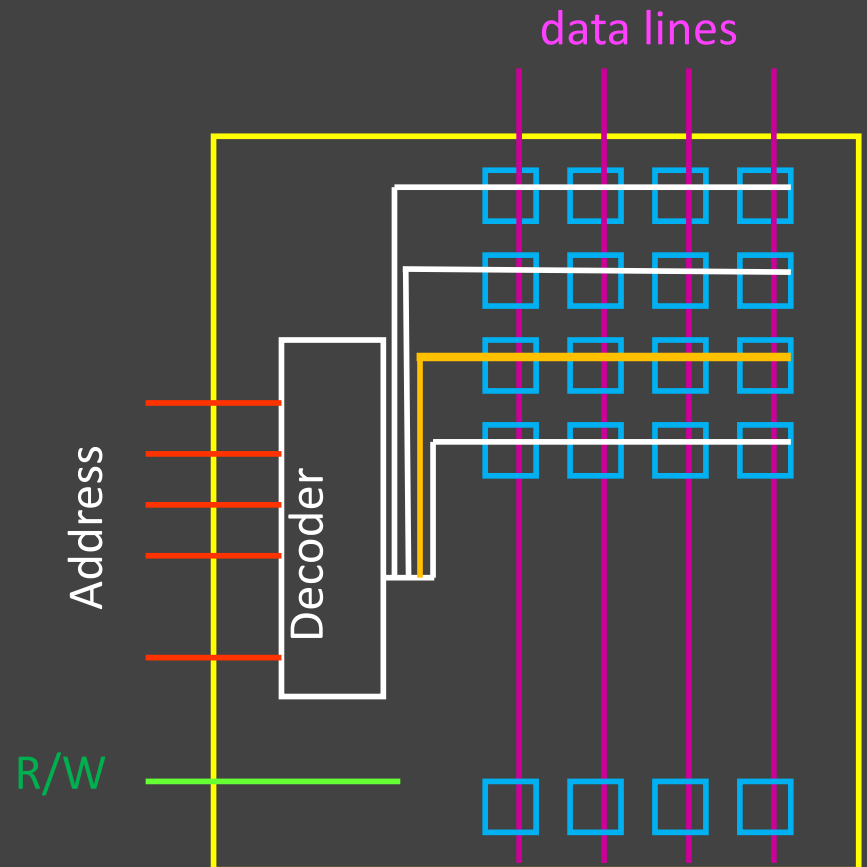
- Storage Cells + bus
- Inputs: Address, Data (for writes)
- Outputs: Data (for reads)
- Also need R/W signal (not shown)



- $N$  address bits  $\rightarrow 2^N$  words total
- $M$  data bits  $\rightarrow$  each word  $M$  bits

# Memory

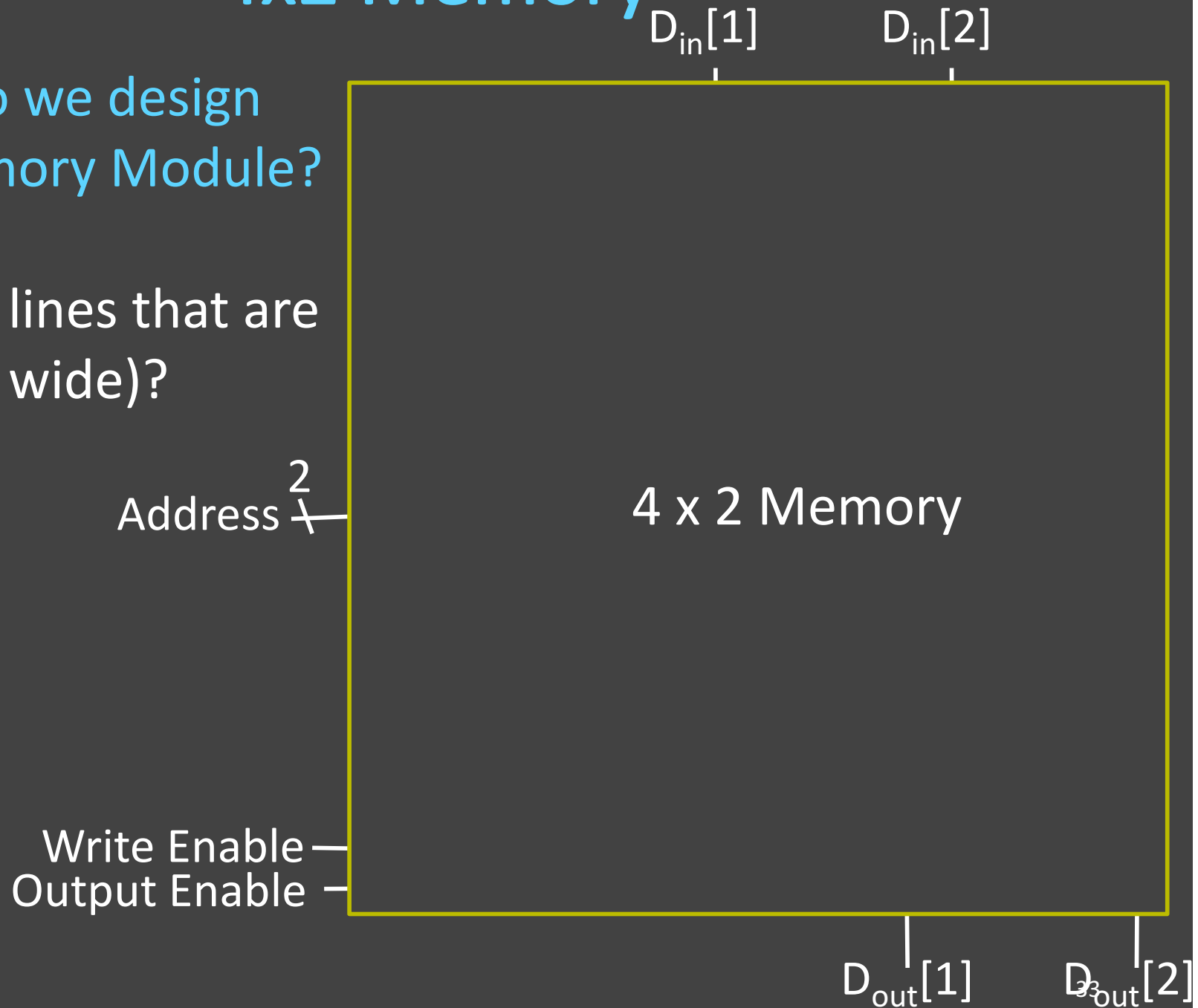
- Storage Cells + bus
- Decoder selects a word line
- R/W selector determines access type
- Word line is then coupled to the data lines



# 4x2 Memory

E.g. How do we design  
a 4 x 2 Memory Module?

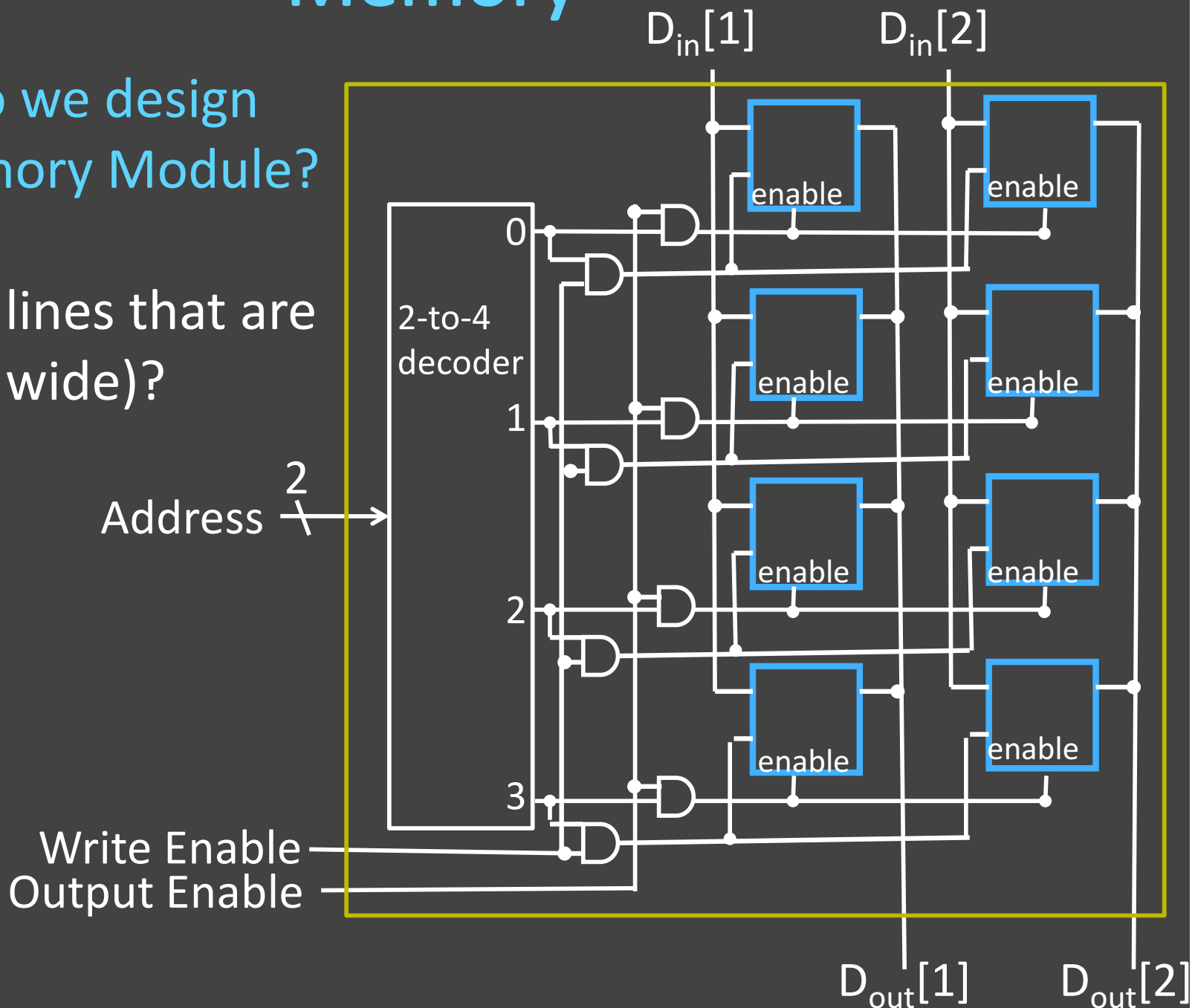
(i.e. 4 word lines that are  
each 2 bits wide)?



# Memory

E.g. How do we design  
a 4 x 2 Memory Module?

(i.e. 4 word lines that are  
each 2 bits wide)?

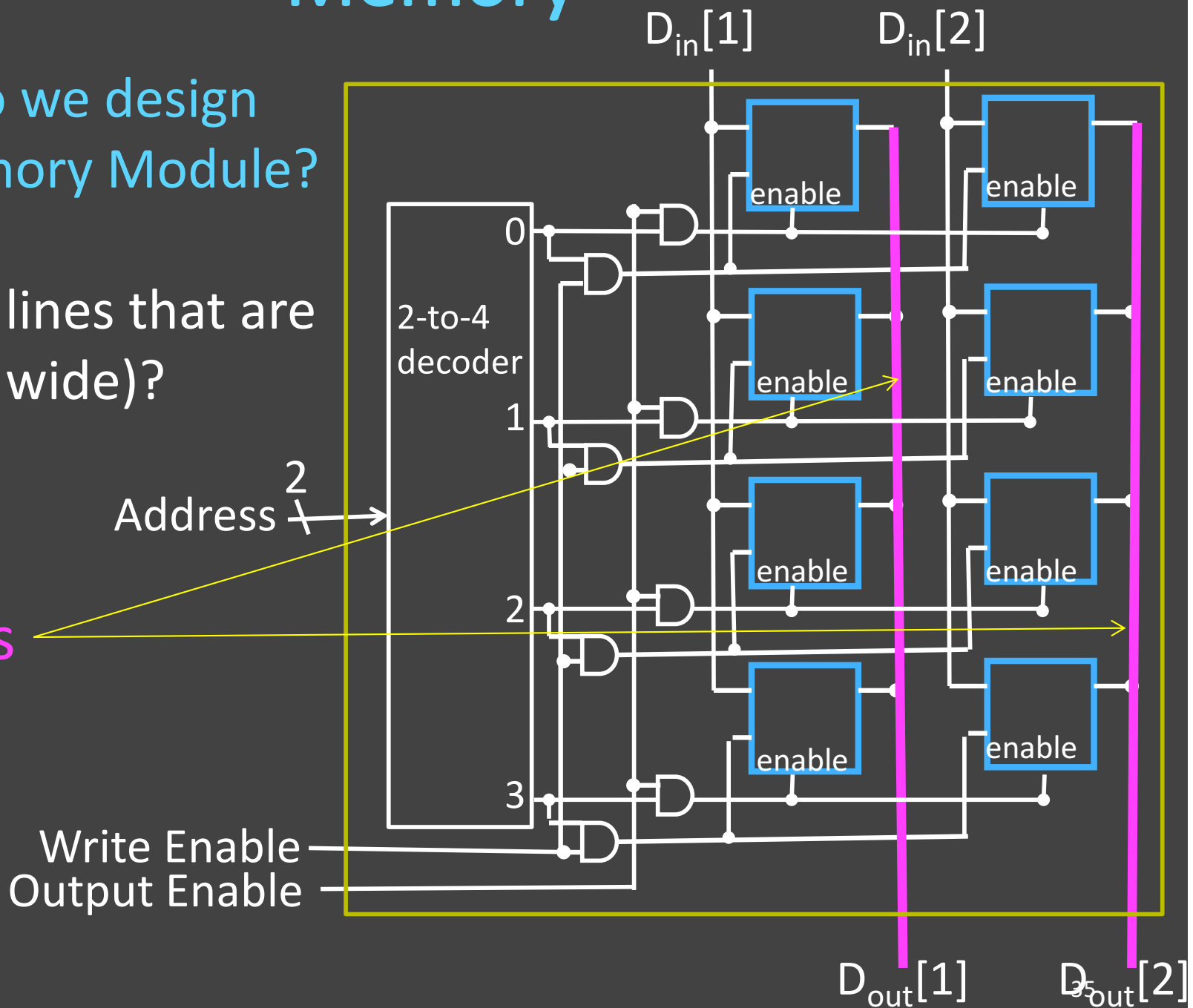


# Memory

E.g. How do we design  
a 4 x 2 Memory Module?

(i.e. 4 word lines that are  
each 2 bits wide)?

Bit lines

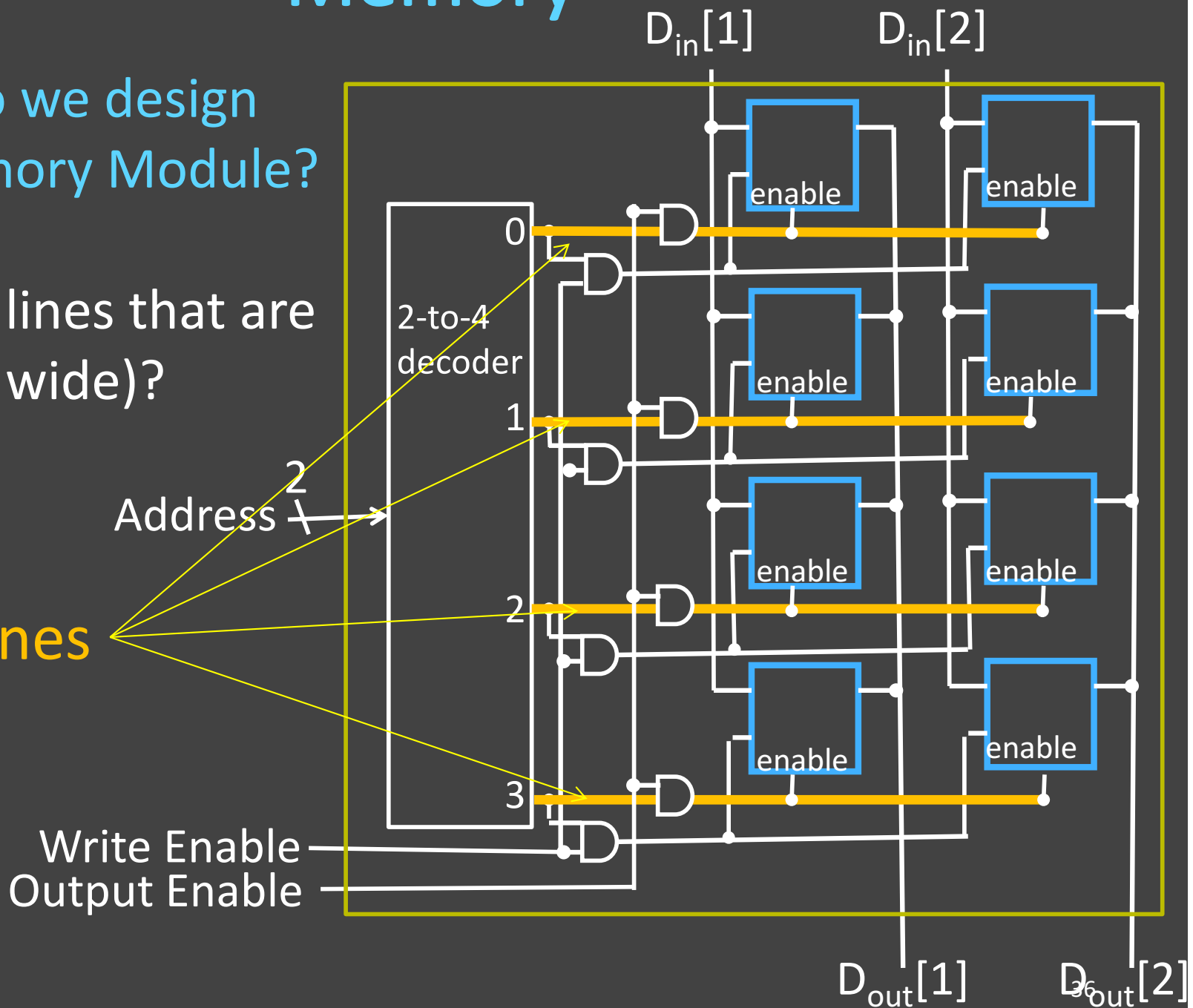


# Memory

E.g. How do we design  
a 4 x 2 Memory Module?

(i.e. 4 word lines that are  
each 2 bits wide)?

Word lines



# iClicker Question

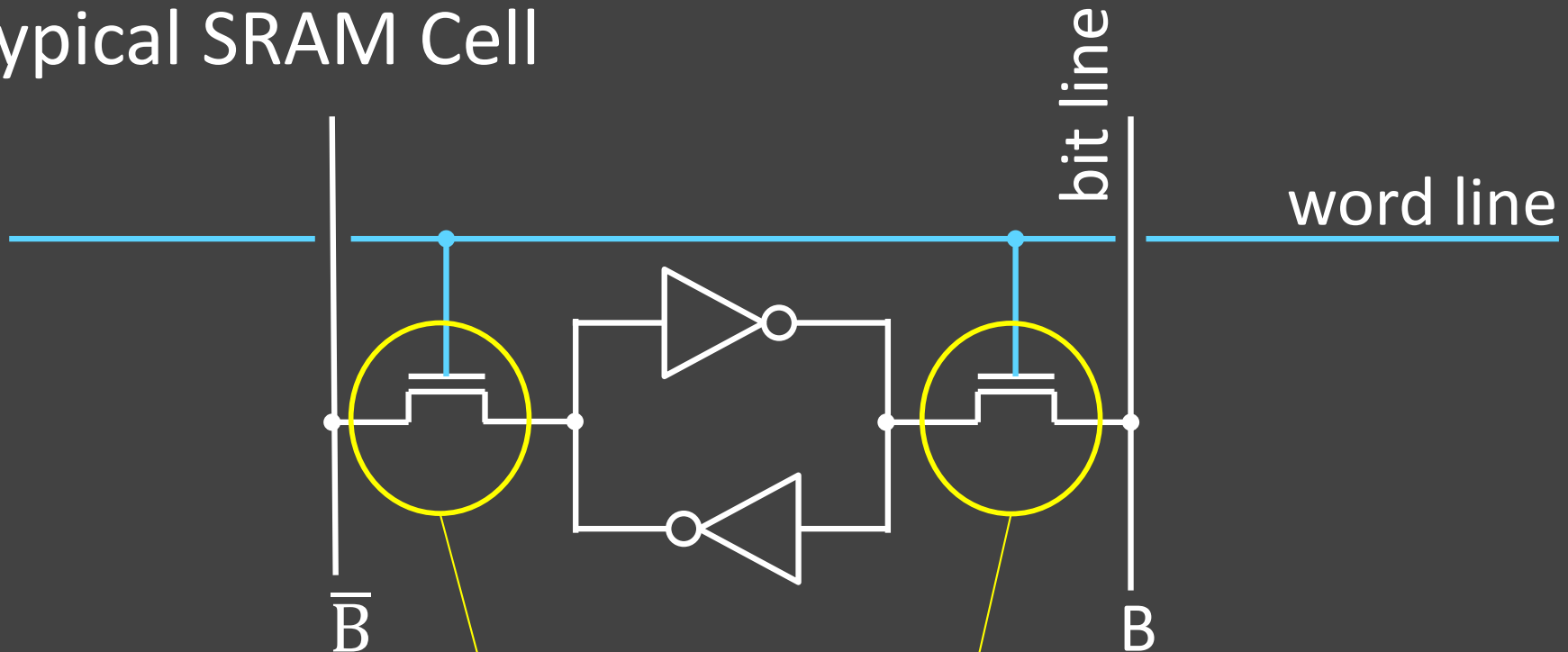
What's your familiarity with memory (SRAM, DRAM)?

- A. I've never heard of any of this.
- B. I've heard the words SRAM and DRAM, but I have no idea what they are.
- C. I know that DRAM means main memory.
- D. I know the difference between SRAM and DRAM and where they are used in a computer system.



# SRAM Cell

## Typical SRAM Cell



Each cell stores one bit, and requires 4 – 8 transistors (6 is typical)

Pass-Through  
Transistors

# SRAM Summary

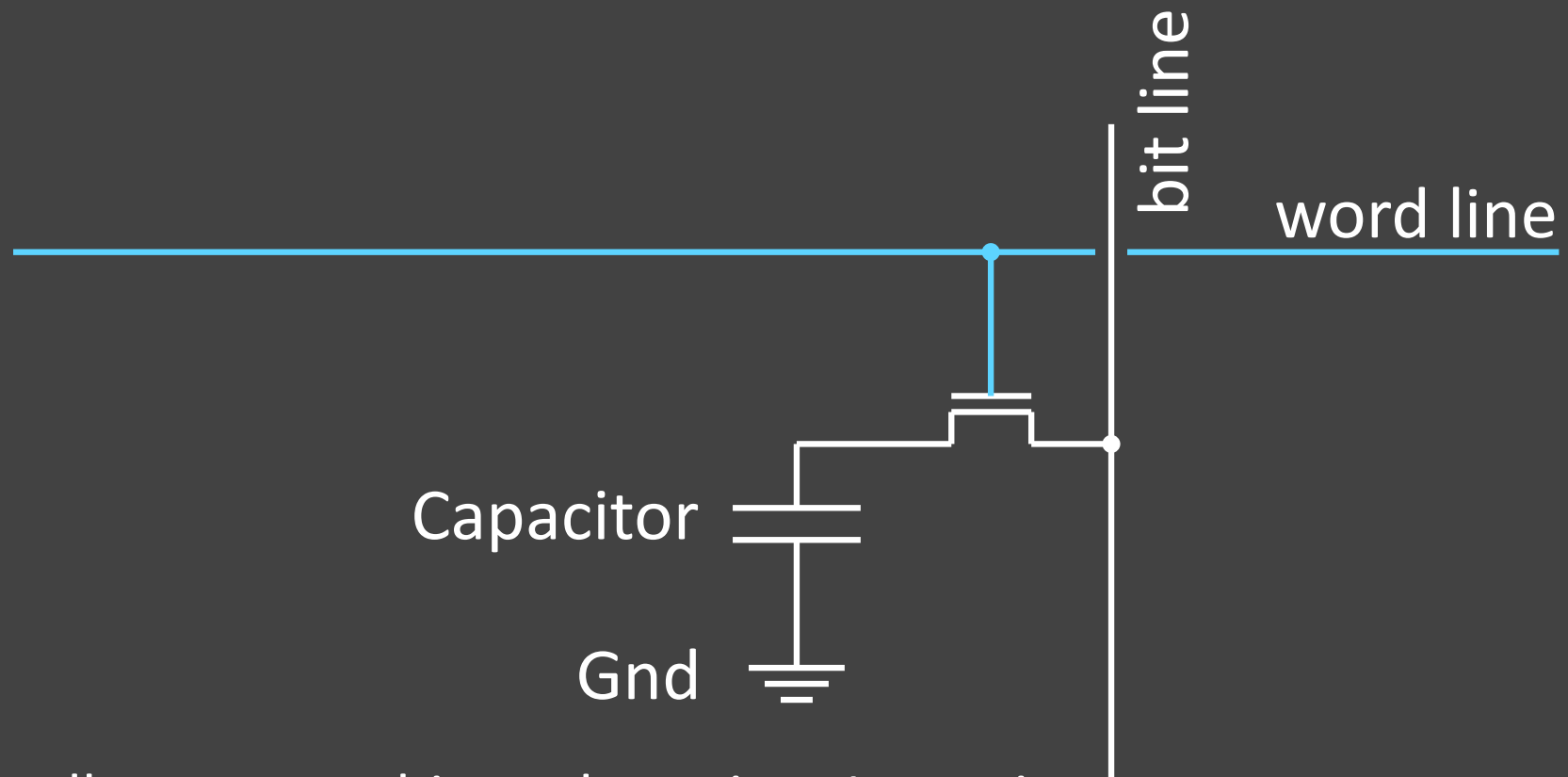
## SRAM

- A few transistors ( $\sim 6$ ) per cell
- Used for working memory (caches)
- But for even higher density...

# Dynamic RAM: DRAM

## Dynamic-RAM (DRAM)

- Data values require constant refresh

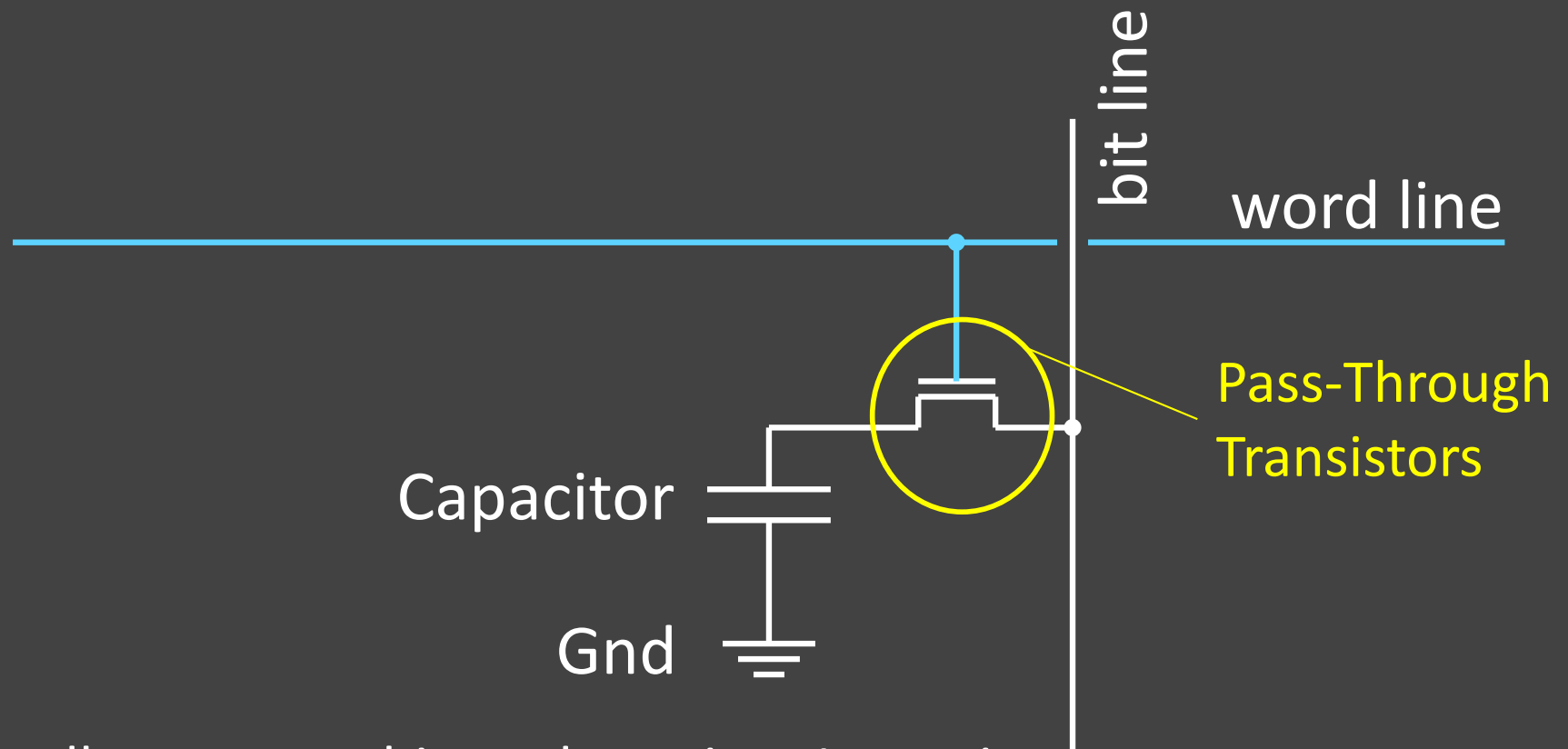


Each cell stores one bit, and requires 1 transistors

# Dynamic RAM: DRAM

## Dynamic-RAM (DRAM)

- Data values require constant refresh



Each cell stores one bit, and requires 1 transistors

# DRAM vs. SRAM

Single transistor vs. many gates

- Denser, cheaper (\$30/1GB vs. \$30/2MB)
- But more complicated, and has analog sensing

Also needs refresh

- Read and write back...
- ...every few milliseconds
- Organized in 2D grid, so can do rows at a time
- Chip can do refresh internally

Hence... slower and energy inefficient

# Memory

## Register File tradeoffs

- + Very fast (a few gate delays for both read and write)
- + Adding extra ports is straightforward
- Expensive, doesn't scale
- Volatile

## Volatile Memory alternatives: SRAM, DRAM, ...

- Slower
- + Cheaper, and scales well
- Volatile

## Non-Volatile Memory (NV-RAM): Flash, EEPROM, ...

- + Scales well
- Limited lifetime; degrades after 100000 to 1M writes

# Goals for Today

## State

- Storing 1 bit
  - Bistable Circuit
  - Set-Reset Latch
  - D Latch
  - D Flip-Flops
- Storing N bits:
  - Registers
  - Memory

## Finite State Machines (FSM)

- Mealy and Moore Machines
- Serial Adder Example

# Finite State Machines

An electronic machine which has

- external inputs
- externally visible outputs
- internal state

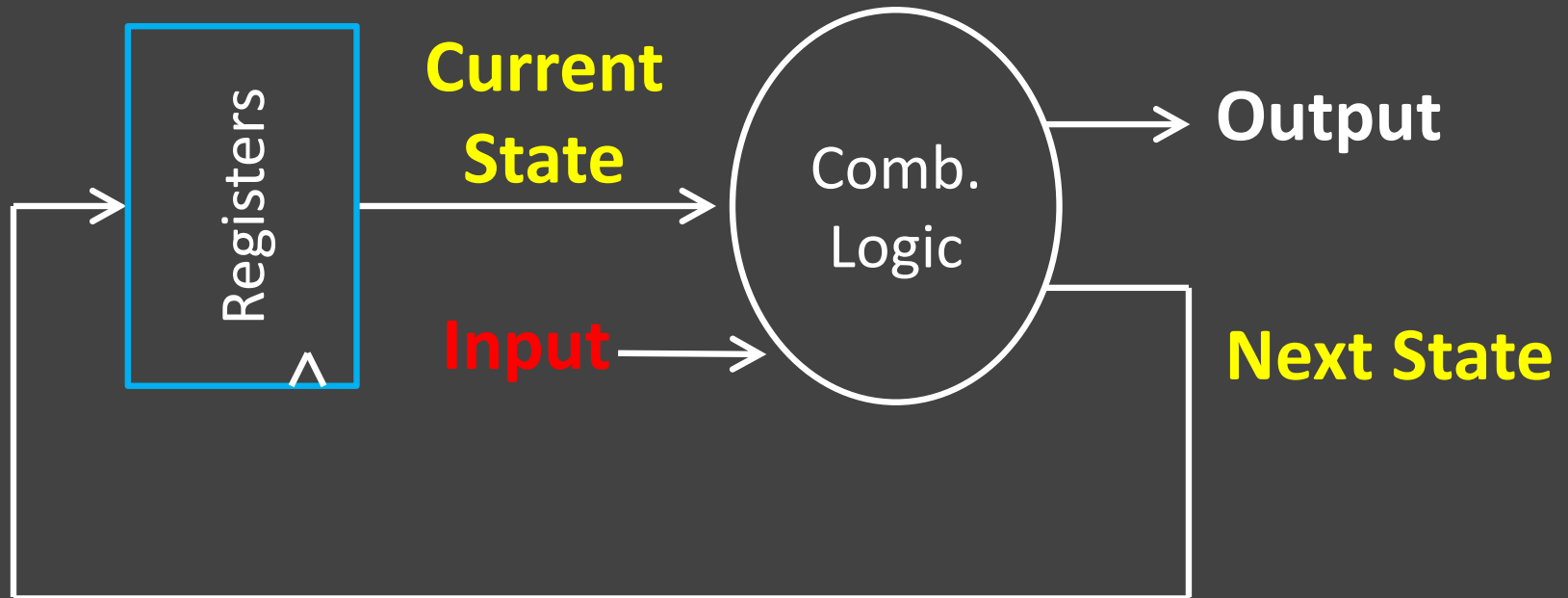
Output and next state depend on

- inputs
- current state



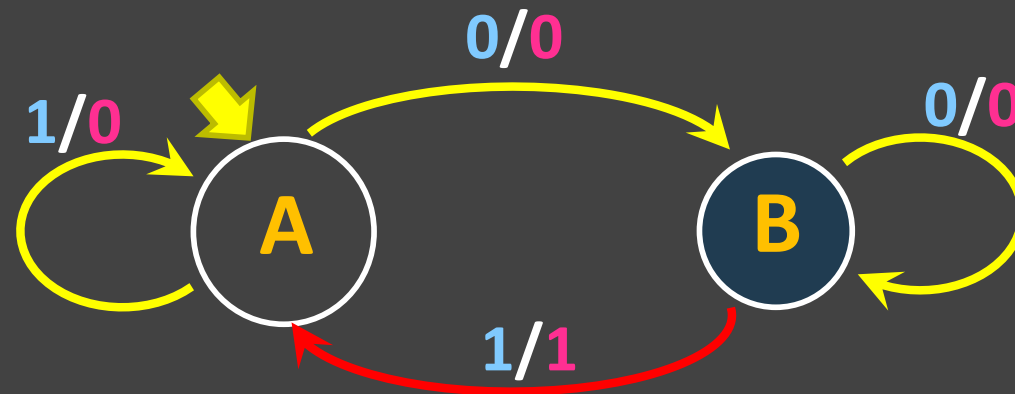
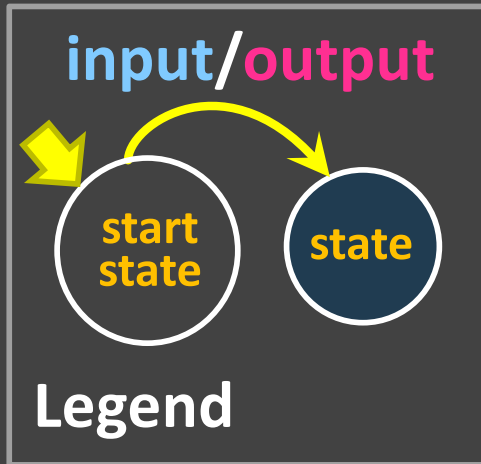
# Automata Model

## Finite State Machine



- inputs from external world
- outputs to external world
- internal state
- combinational logic

# FSM Example



Input: 1 or 0

Output: 1 or 0

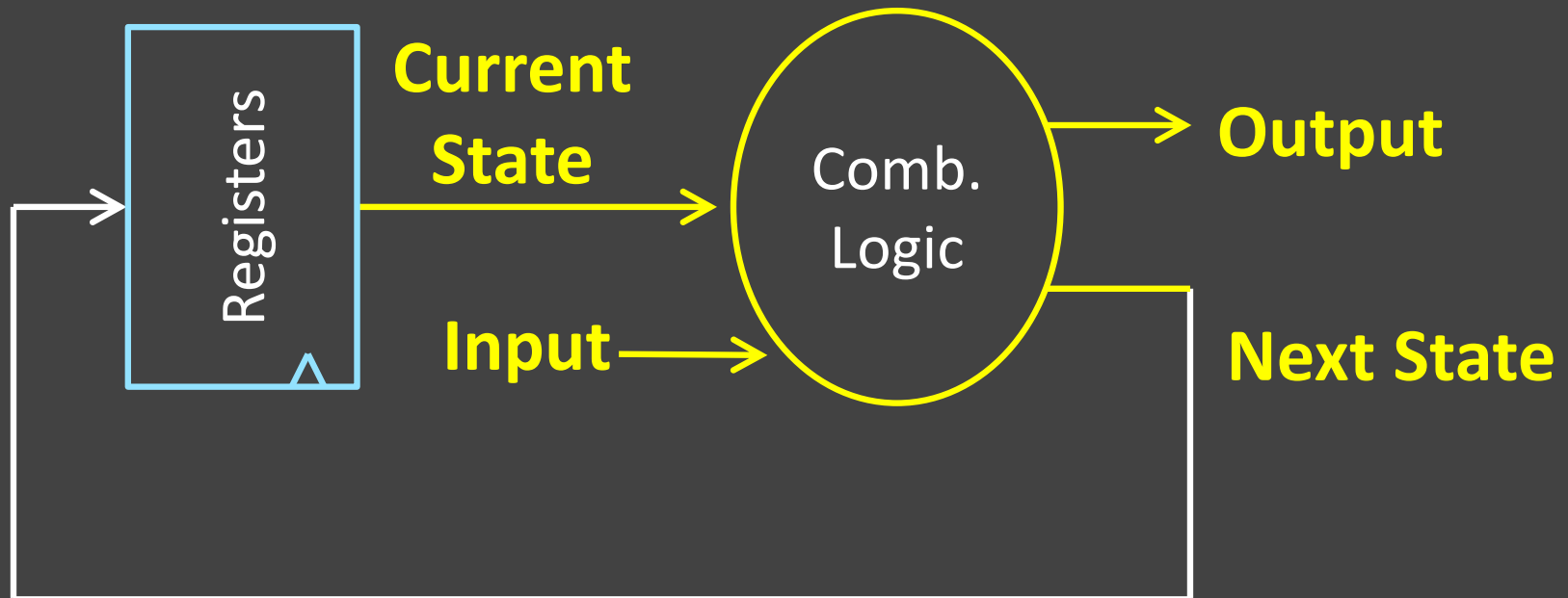
States: A or B

*What input pattern is the FSM “looking for”?*

(Mealy Machine)

# Mealy Machine

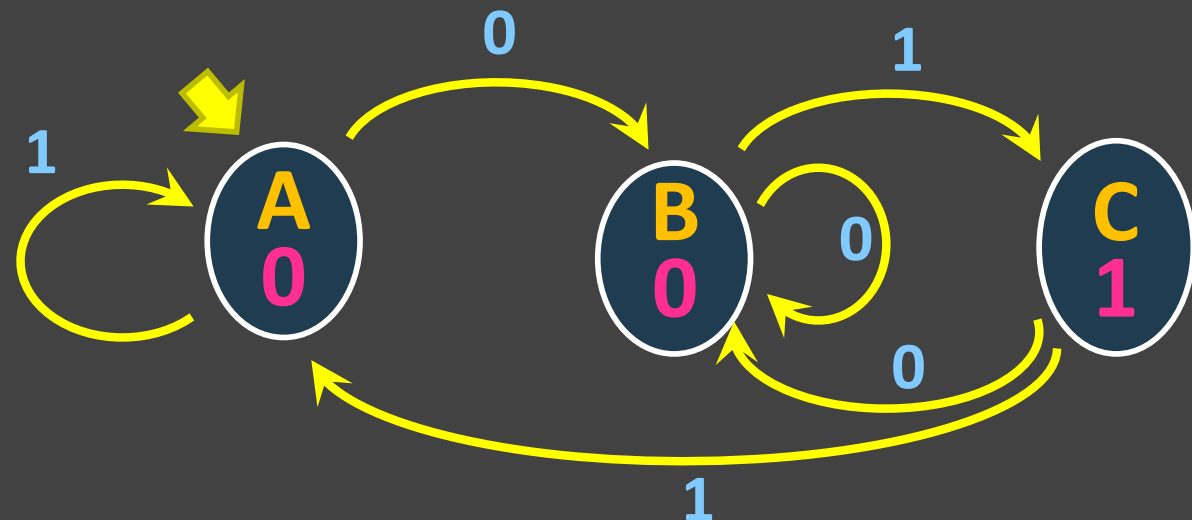
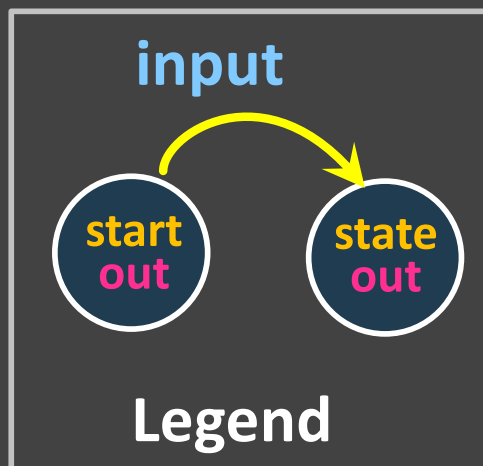
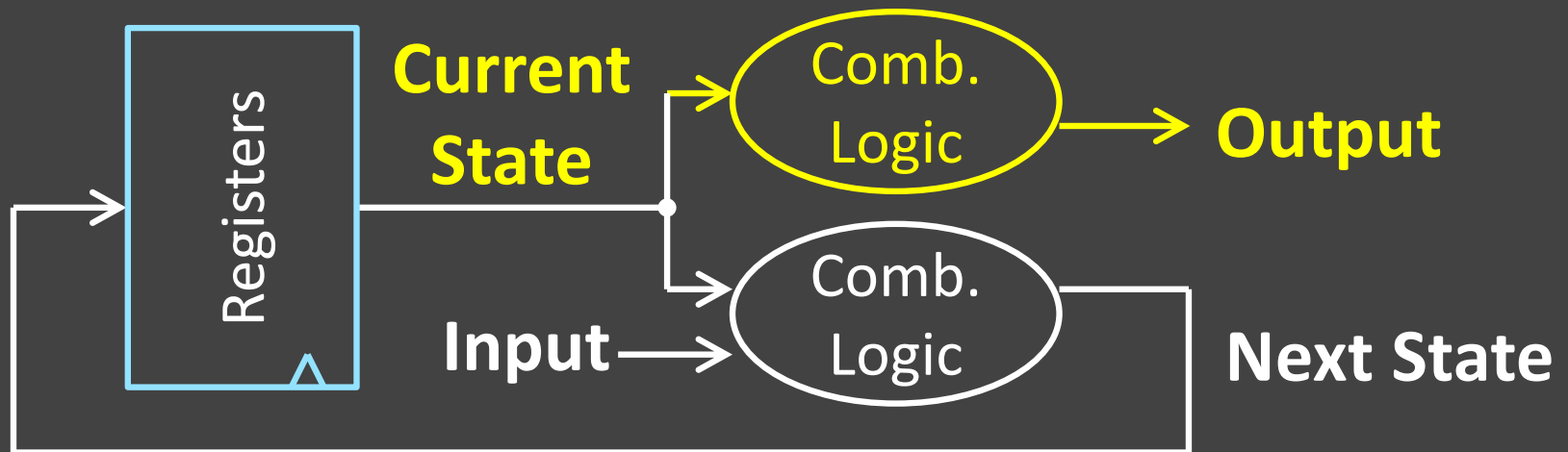
## General Case: Mealy Machine



Outputs and next state depend on both current state and input

# Special Case: Moore Machine

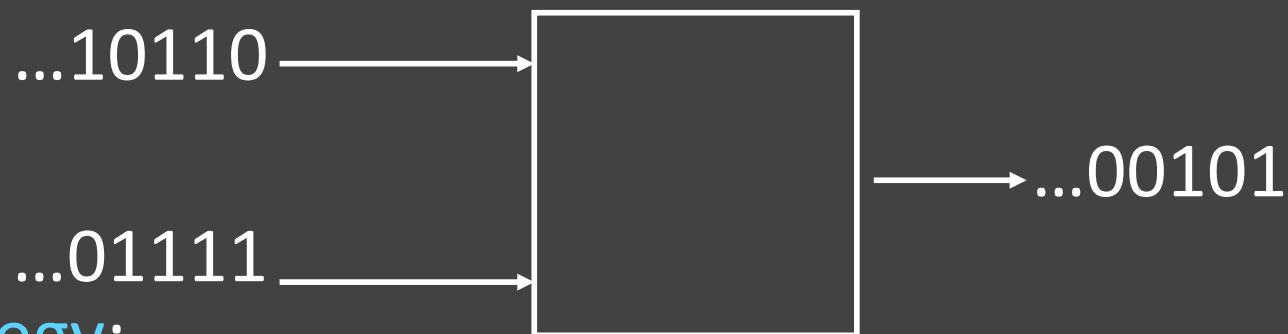
**Outputs** depend only on current state



# Activity: Build a Logic Circuit for a Serial Adder

Add two infinite input bit streams

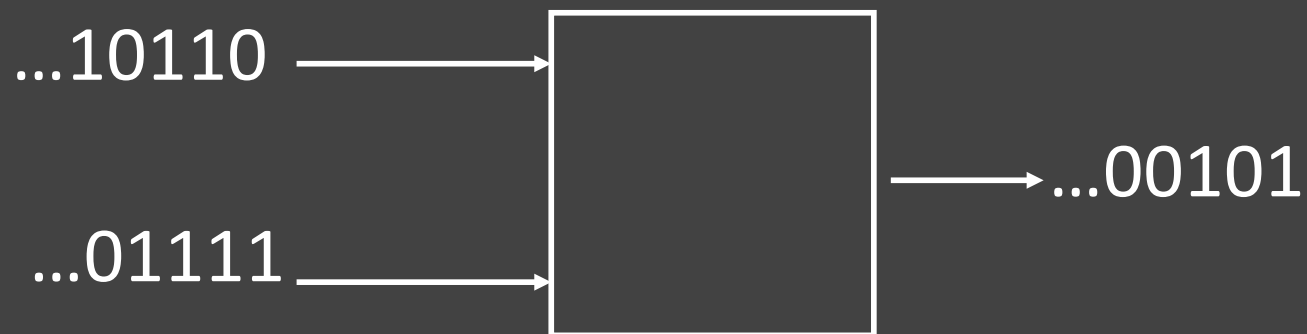
- streams are sent with least-significant-bit (lsb) first
- How many states are needed to represent FSM?
- Draw and Fill in FSM diagram



Strategy:

- (1) Draw a state diagram (e.g. Mealy Machine)
- (2) Write output and next-state tables
- (3) Encode states, inputs, and outputs as bits
- (4) Determine logic equations for next state and outputs

# FSM: State Diagram – start here



\_\_\_ states:

Inputs: ??? and ???

Output: ???

# FSM: State Diagram

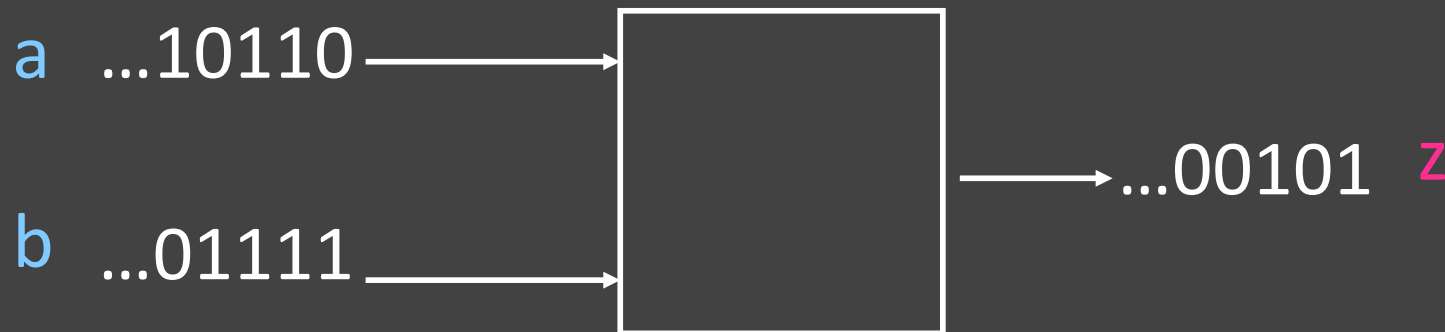
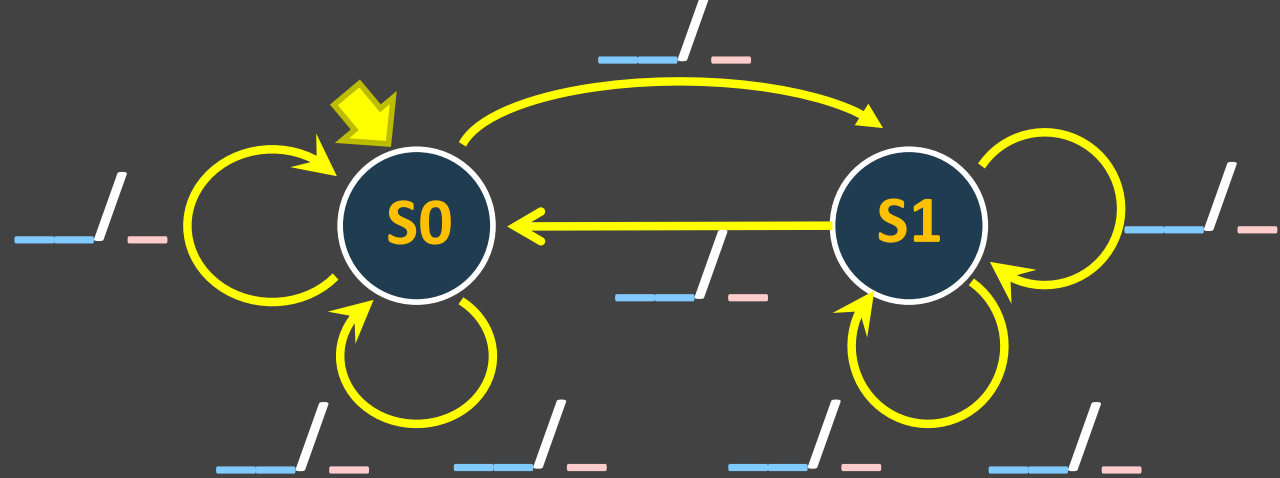


\_\_\_ states:

Inputs: ??? and ???

Output: ???

# FSM: State Diagram



Two states: **S0** (no carry in), **S1** (carry in)

Inputs: **a** and **b**

Output: **z**

- **z** is the sum of inputs **a**, **b**, and carry-in (one bit at a time)
- A carry-out is the next carry-in state.



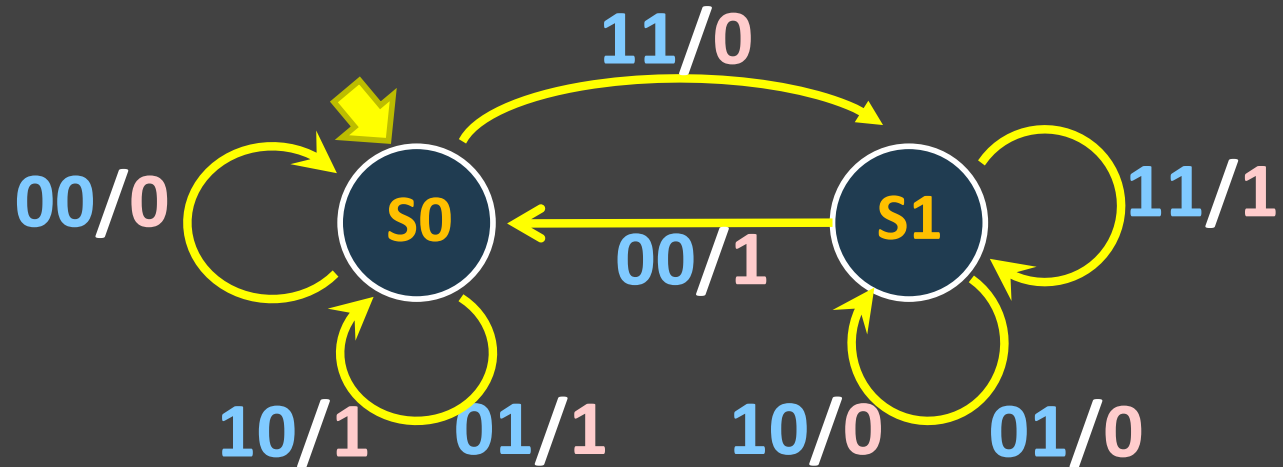
# FSM: State Diagram



??	??	Current state	?	Next state

(2) Write down all input and state combinations

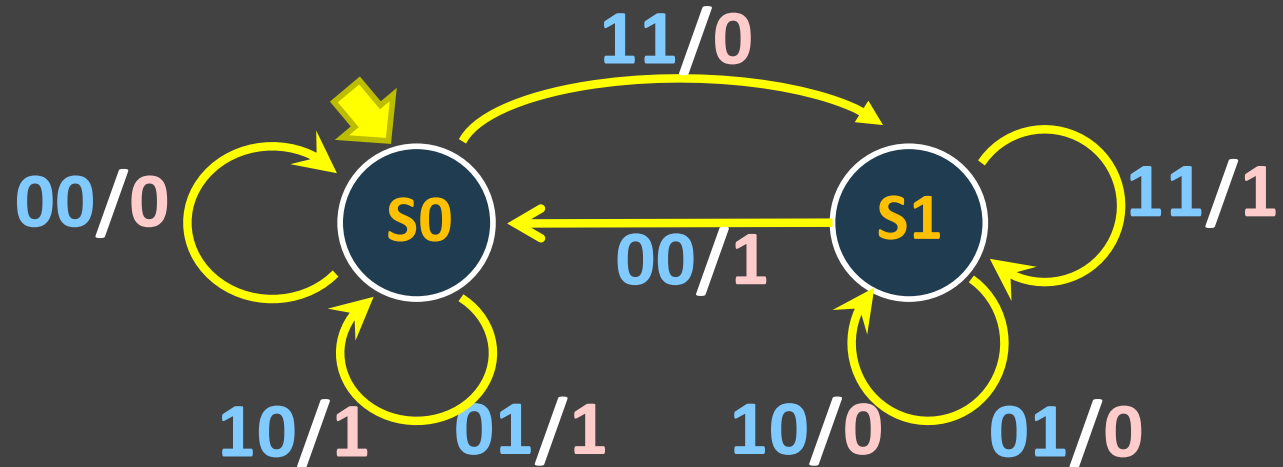
# Serial Adder: State Table



a	b	Current state	z	Next state

(2) Write down all input and state combinations

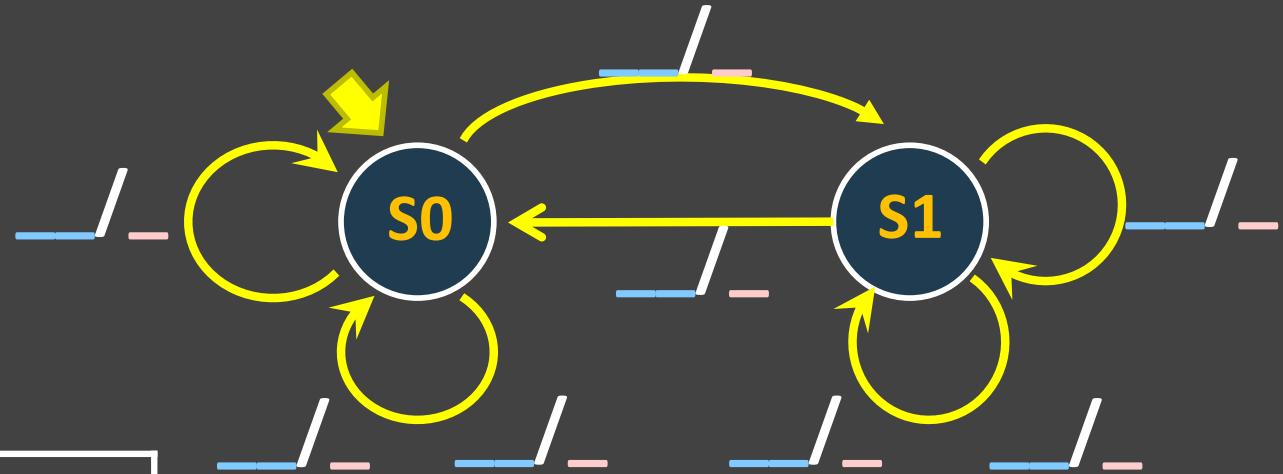
# Serial Adder: State Table



a	b	Current state	z	Next state
0	0	S0	0	S0
0	1	S0	1	S0
1	0	S0	1	S0
1	1	S0	0	S1
0	0	S1	1	S0
0	1	S1	0	S1
1	0	S1	0	S1
1	1	S1	1	S1

(2) Write down all input and state combinations

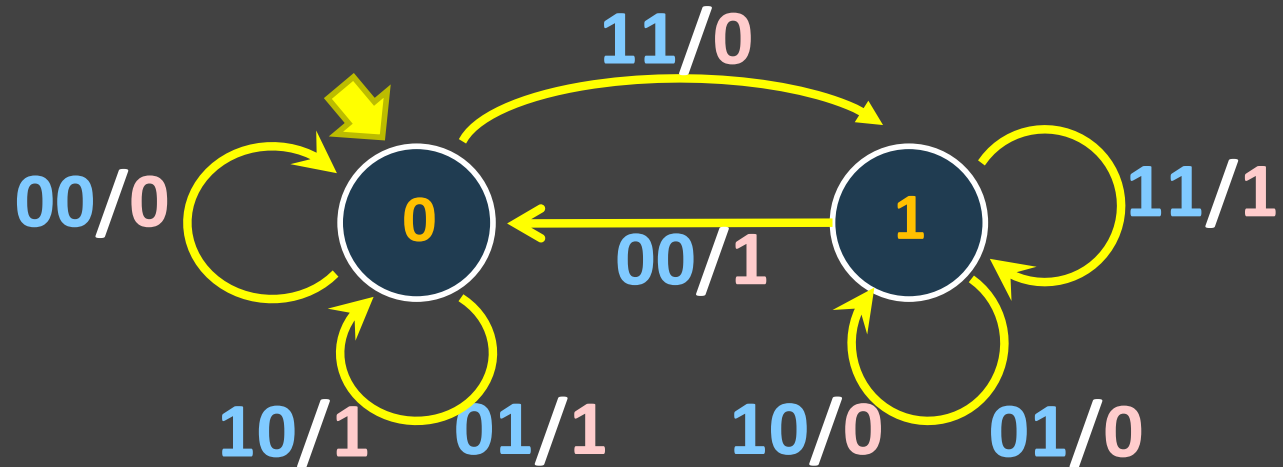
# FSM: State Diagram – start here



??	??	Current state	?	Next state

(3) Encode states, inputs, and outputs as bits

# Serial Adder: State Assignment



a	b	s	z	s'
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

(3) Encode states, inputs, and outputs as bits

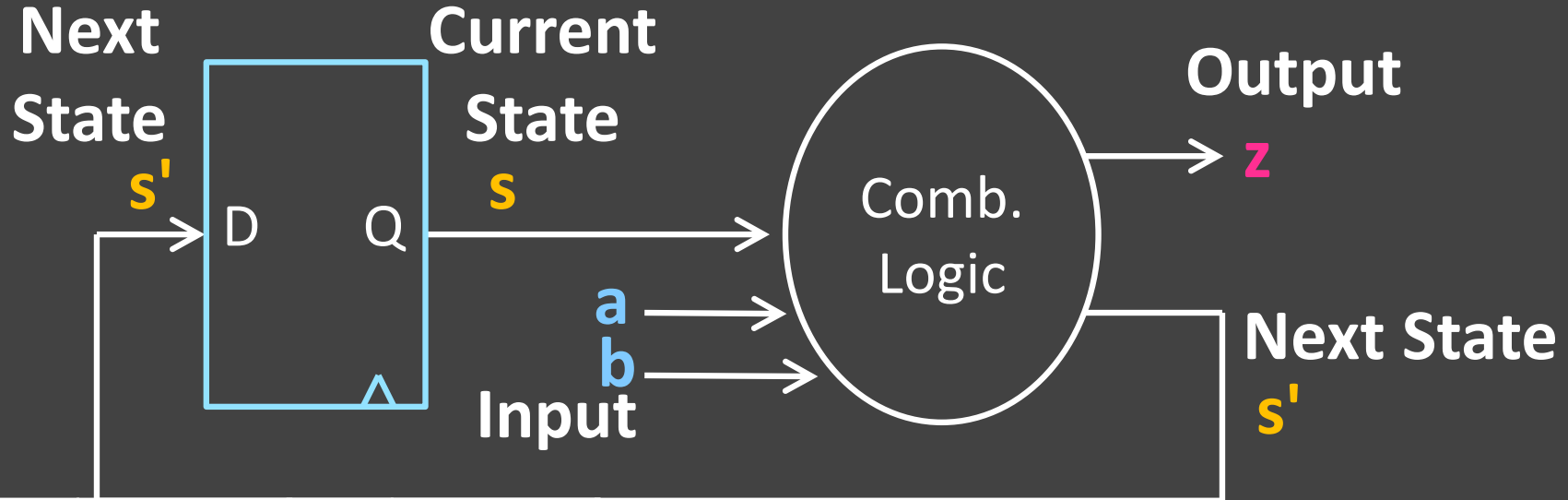
Two states, so 1-bit is sufficient  
(single flip-flop will encode the state)

# FSM: State Diagram

??	??	Current state	?	Next state

(4) Determine logic equations for next state and outputs

# Serial Adder: Circuit



$a$	$b$	$s$	$z$	$s'$
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

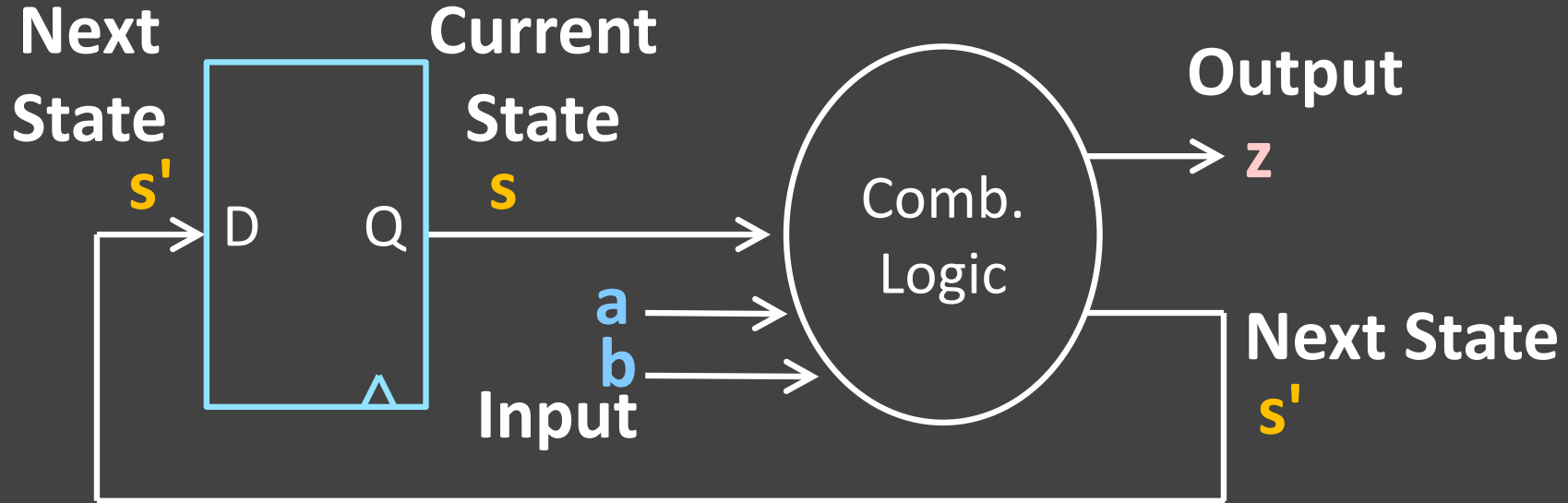
(4) Determine logic equations for next state and outputs

Combinational Logic Equations

$$z = \bar{a}b\bar{s} + a\bar{b}\bar{s} + \bar{a}bs + abs$$

$$s' = ab\bar{s} + \bar{a}bs + \bar{a}\bar{b}s + abs$$

# Sequential Logic Circuits



$$z = \bar{a}b\bar{s} + a\bar{b}\bar{s} + \bar{a}\bar{b}s + abs$$

$$s' = ab\bar{s} + \bar{a}bs + a\bar{b}s + abs$$

## Strategy:

- (1) Draw a state diagram (e.g. Mealy Machine)
- (2) Write output and next-state tables
- (3) Encode states, inputs, and outputs as bits
- (4) Determine logic equations for next state and outputs



# Summary

We can now store data values

- Stateful circuit elements (D Flip Flops, Registers, ...)
- Clock synchronizes state changes
- State Machines or Ad-Hoc Circuits