# Gates and Logic: From Transistors to Logic Gates and Logic Circuits

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**CS 3410**

Computer Science

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The slides are the product of many rounds of teaching CS 3410 by Professors Weatherspoon, Bala, Bracy, and Sirer.

See corresponding Chapter in your zybook

## Goals for Today

From Switches to Logic Gates to Logic Circuits

Transistors (electronic switch)

Logic Gates

- Truth Tables

Logic  Circuits

- Identity Laws
- From Truth Tables to Circuits (Sum of Products)

Logic Circuit Minimization

- Algebraic Manipulations
- Karnaugh Maps

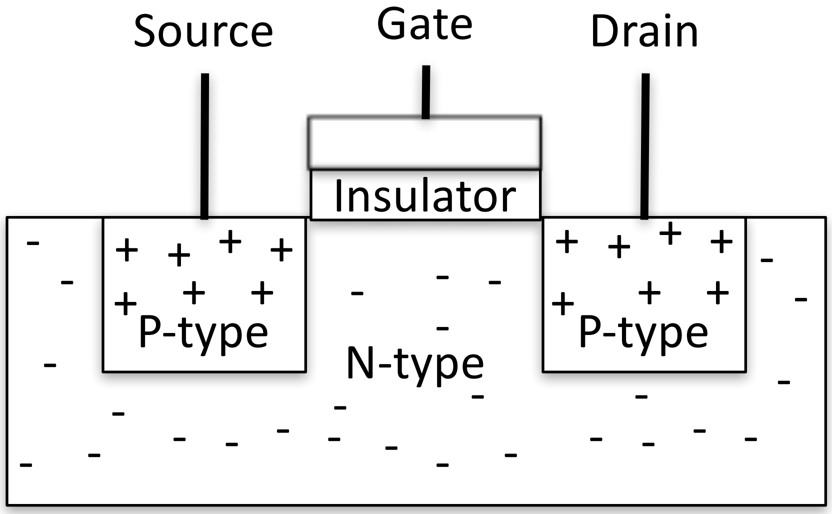
## Silicon Valley & the Semiconductor Industry

Transistors:

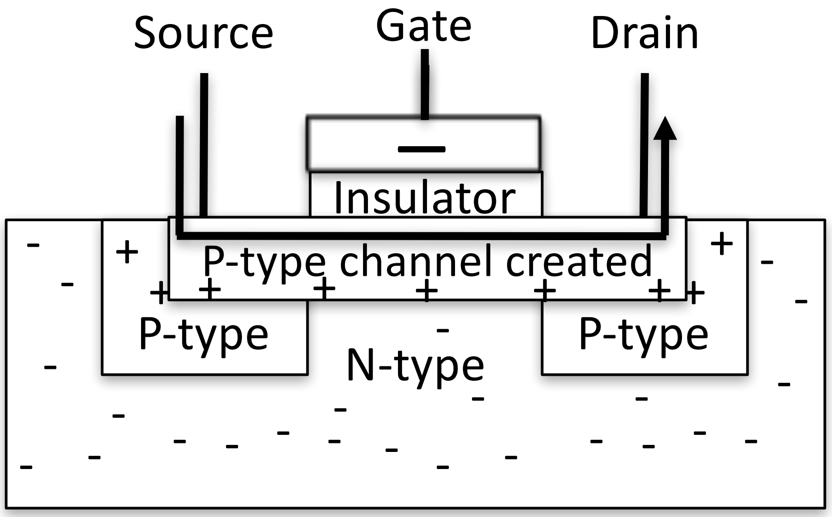
- Youtube video from last week
  https://www.youtube.com/watch?v=IcrBqCFLHIY

- Show some Transistor, Fab, Wafer photos

# Transistors 101



**Off**



**On**

**N-Type Silicon:** negative free-carriers (electrons)

**P-Type Silicon:** positive free-carriers (holes)

**P-Transistor:** negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain

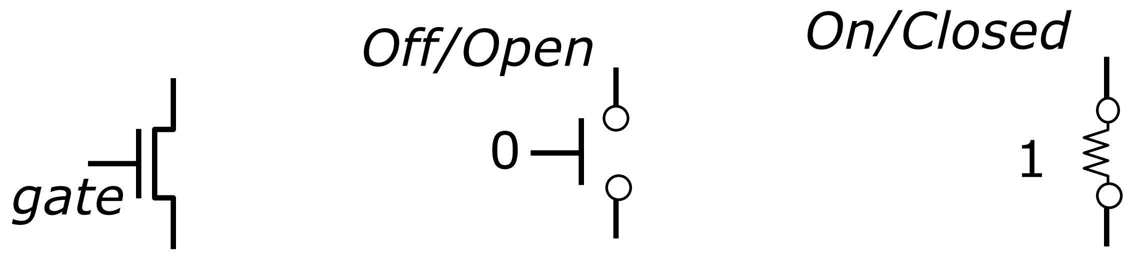
**N-Transistor:** works the opposite way

Metal-Oxide Semiconductor (Gate-Insulator-Silicon)

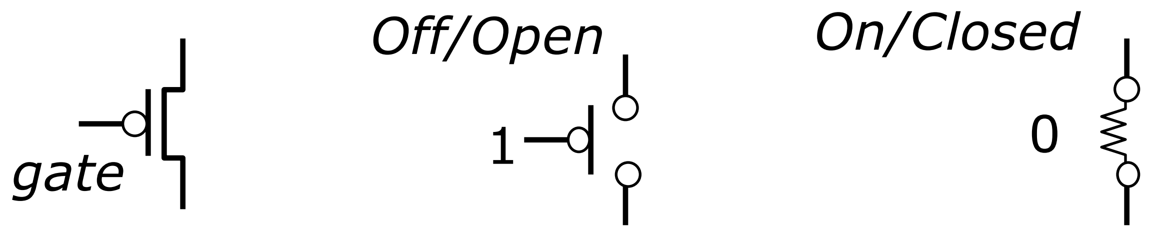
Complementary MOS = **CMOS** technology uses both p- & n-type transistors

## CMOS Notation

N-type



P-type



Gate input controls whether current can flow between the other two terminals or not.

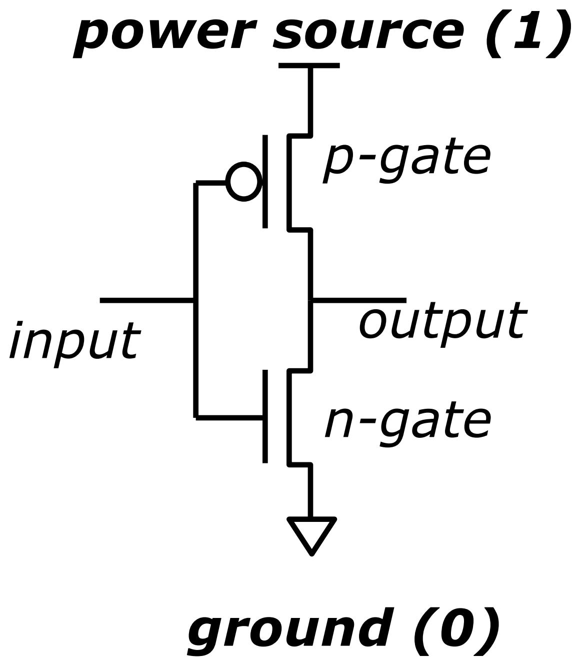
*Hint:* the “o” bubble of the p-type tells you that this gate wants a 0 to be turned on

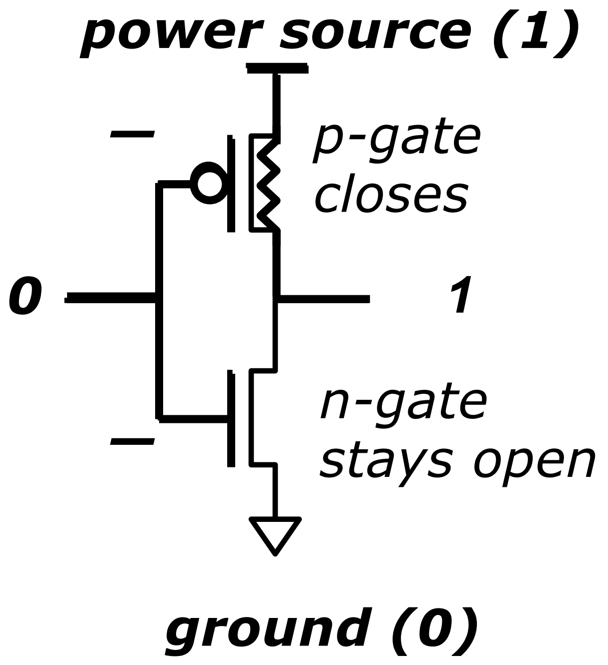
## 2-Transistor Combination: NOT

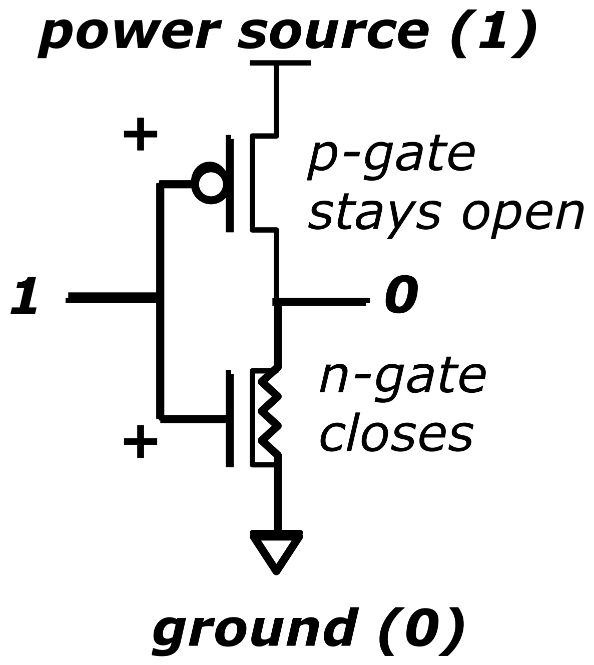
Logic gates are constructed by combining transistors in complementary arrangements

Combine p&n transistors to make a NOT gate:

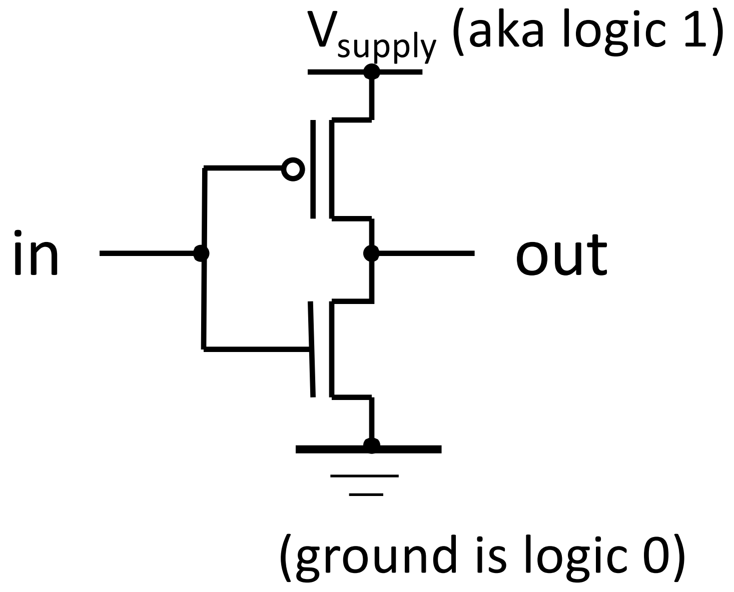
*CMOS Inverter :*



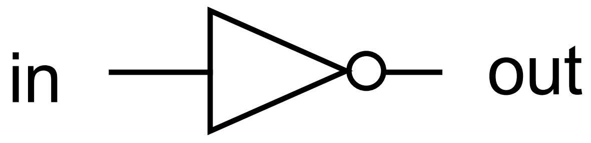




# Inverter



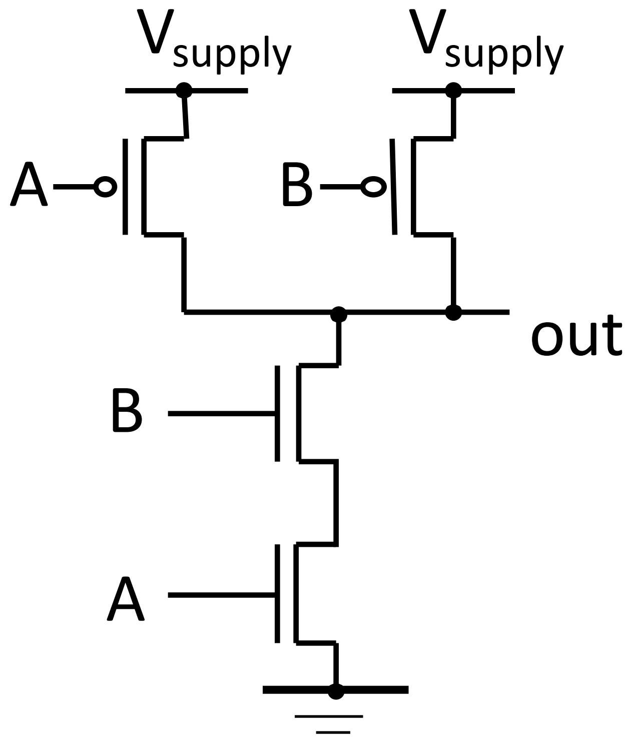
- Function: NOT
- Symbol:



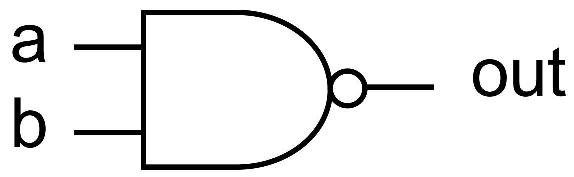
- Truth Table:

| In | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

## Activity #1: Which Gate is this?



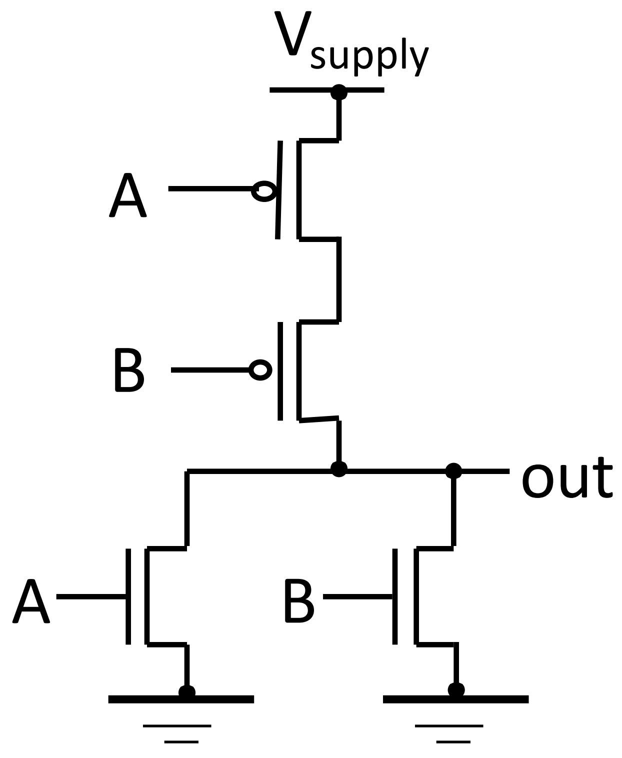
- Function:
- Symbol:



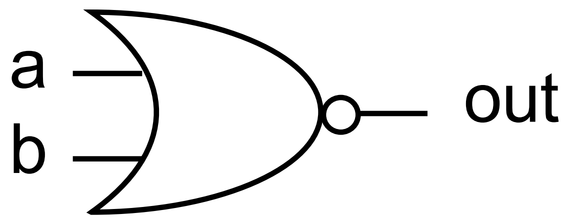
- Truth Table:

| A | B | out |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |

# NOR Gate



- Function: NOR
- Symbol:



- Truth Table:

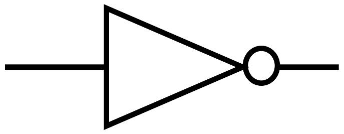
| A | B | out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

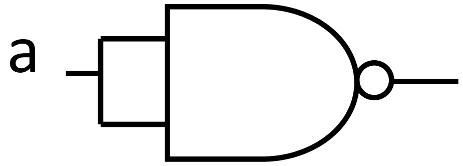
# Building Functions

NAND and NOR are universal

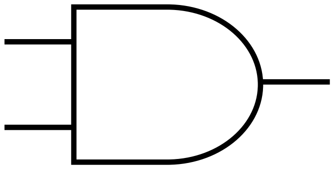
- Can implement ***any*** function with NAND or just NOR gates
- useful for manufacturing

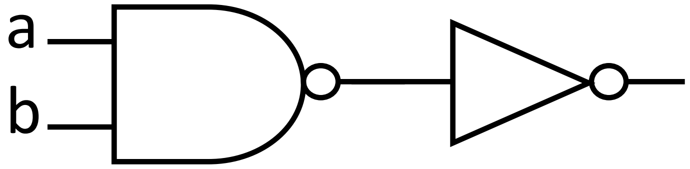
NOT:



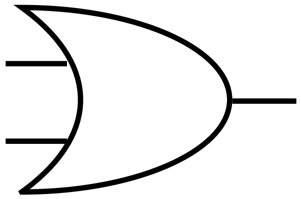


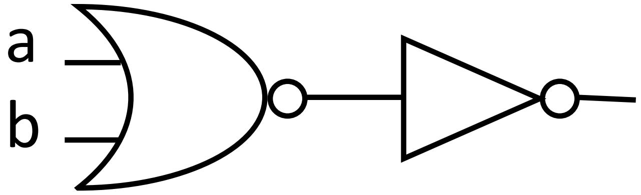
AND:





OR:

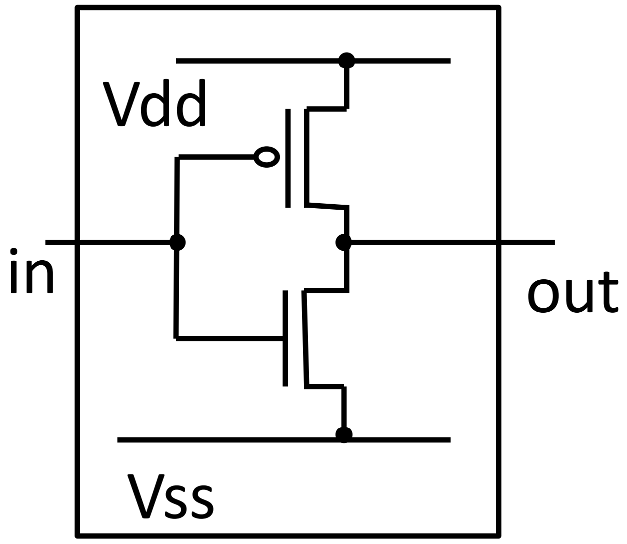


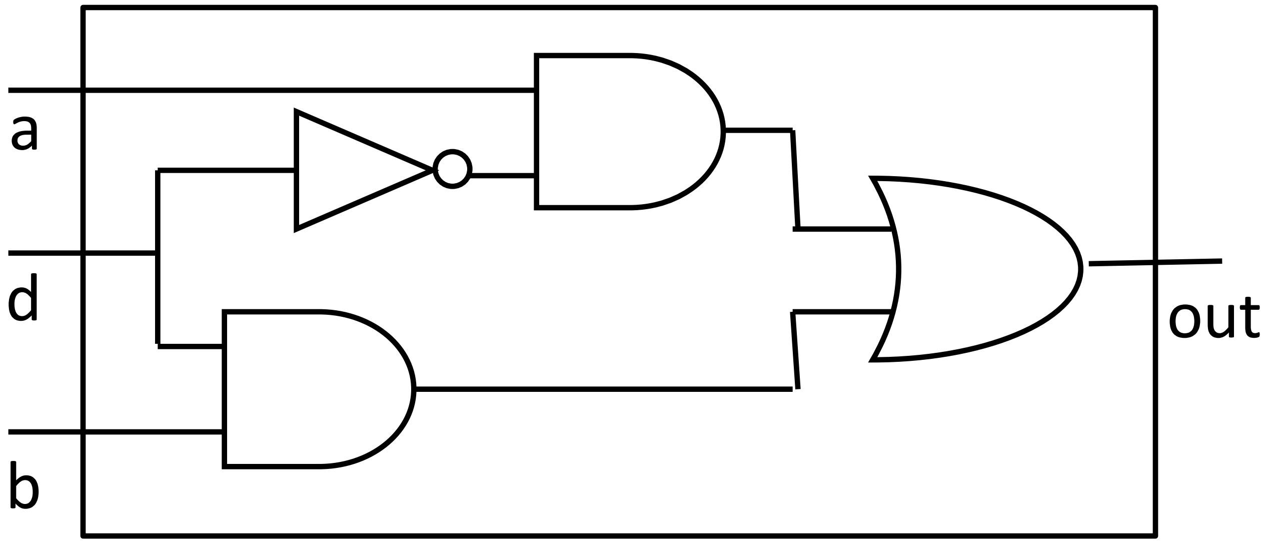


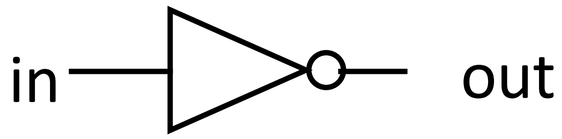
# Big Picture: Abstraction

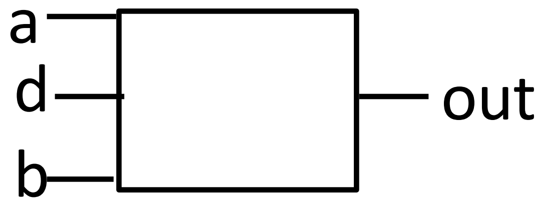
Hide complexity through simple abstractions

- Simplicity
  - Box diagram represents inputs and outputs
- Complexity
  - Hides underlying NMOS- and PMOS-transistors and atomic interactions









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Logic Circuit Minimization

- Algebraic Manipulations
- Karnaugh Maps

# Logic Gates

- Digital circuit that either allows signal to pass through it or not
- Used to build logic functions
- Seven basic logic gates:

  **AND,**

  **OR**,

  **NOT**,

  **NAND** (not AND),

  **NOR** (not OR),

  **XOR**

  **XNOR** (not XOR)

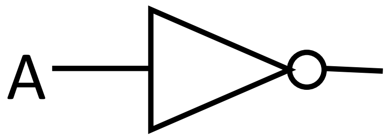
**George Boole,(1815–1864)**

Did you know?

George Boole Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.

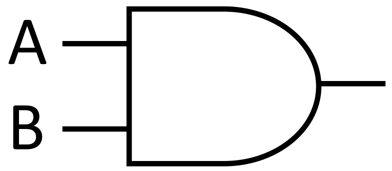
# Logic Gates: Names, Symbols, Truth Tables

NOT:



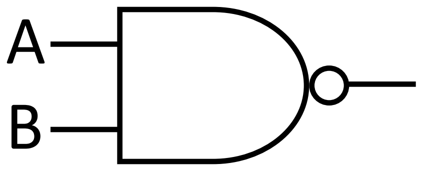
| A | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

AND:



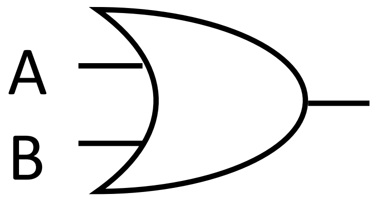
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

NAND:



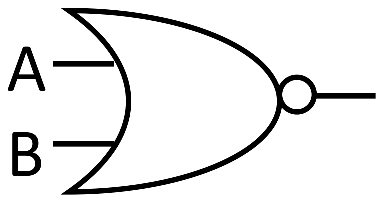
| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

OR:



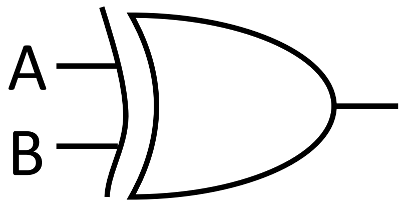
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NOR:



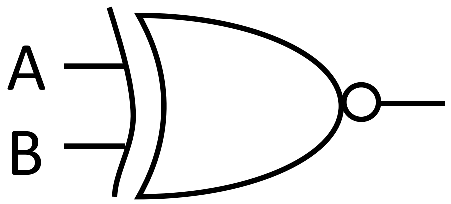
| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

XOR:.



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

XNOR:



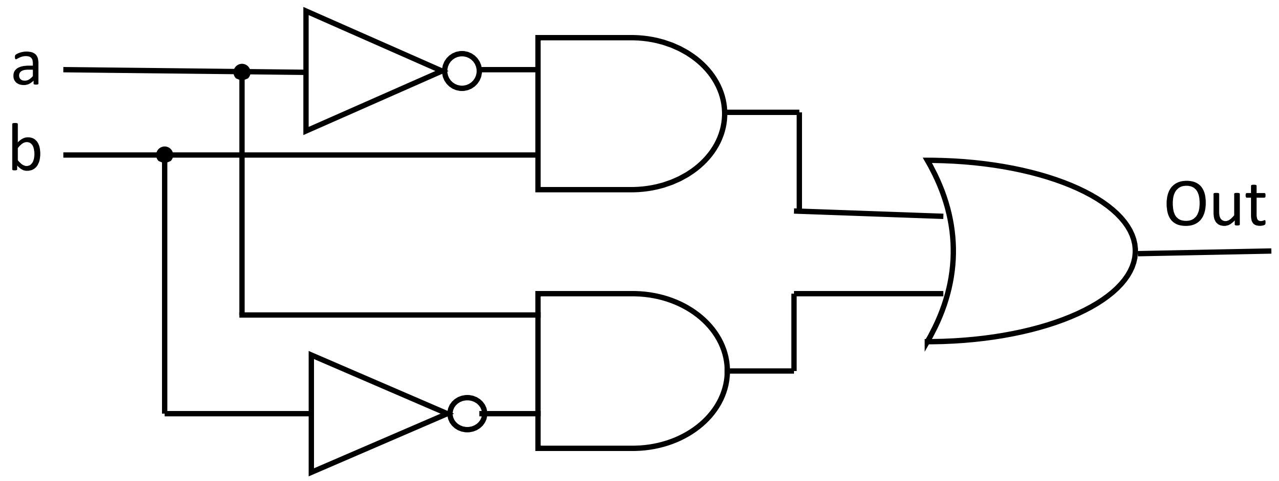
| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Activity #2: Logic Gates

Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.

What does the logic circuit do?

| a | b | Out |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

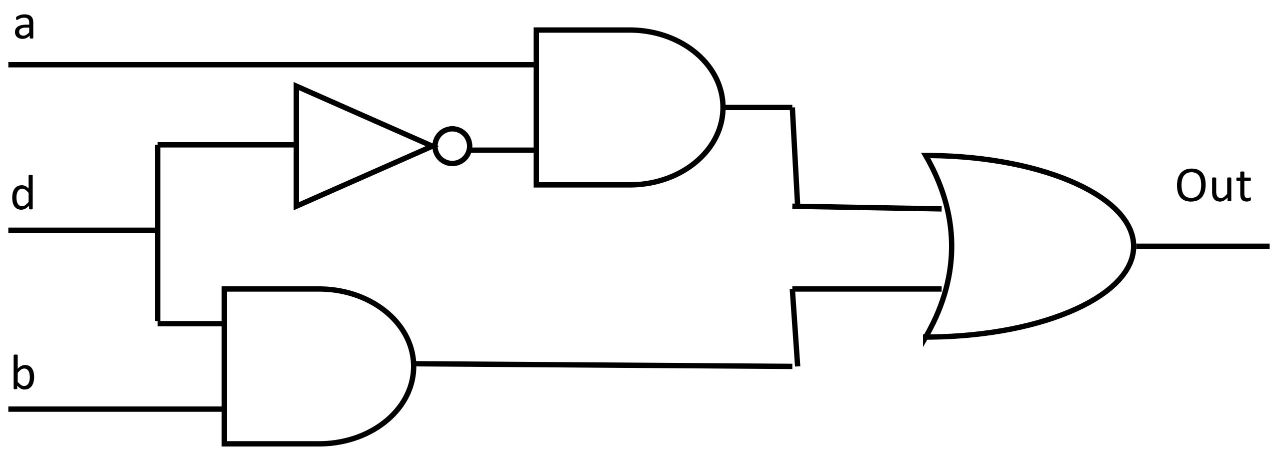


# Activity #3: Logic Gates

Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.

What does the logic circuit do?

| a | b | d | Out |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |



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## Next Goal

Given a Logic function → create a Logic Circuit that implements the Logic Function…

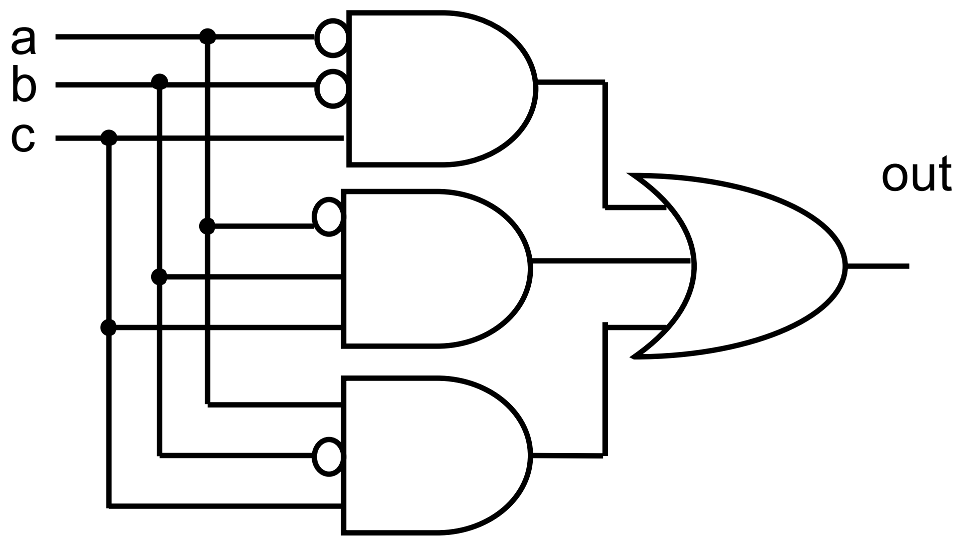
# Logic Implementation

How to implement a desired logic function?

|  | a | b | c | out | minterm |
|---|---|---|---|---|---|
|  | **0** | **0** | **0** | **0** | $\bar{a}\ \bar{b}\ \bar{c}$ |
| ⇨ | **0** | **0** | **1** | **1** | $\bar{a}\ \bar{b}\ c$ |
|  | **0** | **1** | **0** | **0** | $\bar{a}\ b\ \bar{c}$ |
| ⇨ | **0** | **1** | **1** | **1** | $\bar{a}\ b\ c$ |
|  | **1** | **0** | **0** | **0** | $a\ \bar{b}\ \bar{c}$ |
| ⇨ | **1** | **0** | **1** | **1** | $a\ \bar{b}\ c$ |
|  | **1** | **1** | **0** | **0** | $a\ b\ \bar{c}$ |
|  | **1** | **1** | **1** | **0** | $a\ b\ c$ |

1) Write minterms
2) Write sum of products:
   OR of all minterms where out=1

$$\text{out} = \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}c$$



*Any* combinational circuit *can be* implemented in two levels of logic (ignoring inverters)

# Logic Equations

NOT: $= \bar{a}$ $= !a$ $= \neg a$

AND: $= a \cdot b$ $= a \mathbin{\&} b$ $= a \wedge b$

NAND: $\overline{(a \bullet b)}$ $= !(a \mathbin{\&} b)$ $= \neg (a \wedge b)$

OR: $= a + b$ $= a \mid b$ $= a \vee b$

NOR: $\overline{(a + b)}$ $= !(a \mid b)$ $= \neg (a \vee b)$

XOR: $= a \oplus b = a\bar{b} + \bar{a}b$

XNOR: $\overline{(a \oplus b)}$ $= \overline{a\bar{b} + \bar{a}b}$

## Logic Equations

- Constants: true = 1, false = 0
- Variables: a, b, out, ...
- Operators (above): AND, OR, NOT, etc.

## Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$a + 0 = a$

$a + 1 = 1$

$a + \bar{a} = 1$

$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot \bar{a} = 0$

# Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

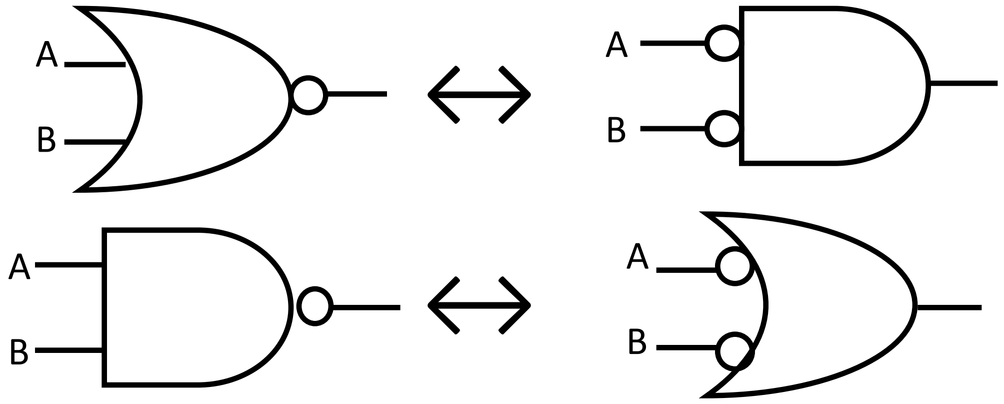
$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

a + a b = a

a (b+c) = ab + ac

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$



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## Next Goal

Implement the Logic Function…

*with the minimum number of logic gates*

Fewer gates: A cheaper ($$$) circuit!

How to standardize this minimizing?

## Activity #4: Logic Minimization

$a + 0 = a$

$a + 1 = 1$

$a + \bar{a} = 1$

$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot \bar{a} = 0$

$a + a\,b = a$

$a\,(b+c) = ab + ac$

$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$

Minimize this logic equation:

$(a+b)(a+c) =$

## Checking Equality w/Truth Tables

circuits ↔ truth tables ↔equations

- Example: (a+b)(a+c) = a + bc

| a | b | c | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | | | | |
| 0 | 0 | 1 | | | | | |
| 0 | 1 | 0 | | | | | |
| 0 | 1 | 1 | | | | | |
| 1 | 0 | 0 | | | | | |
| 1 | 0 | 1 | | | | | |
| 1 | 1 | 0 | | | | | |
| 1 | 1 | 1 | | | | | |

## Minimization in Practice

How does one find the most efficient equation?

- Manipulate algebraically until…?
- Use Karnaugh Maps (optimize visually)
- Use a software optimizer

For large circuits

- Decomposition & reuse of building blocks

# Building a Karnaugh Map

| a | b | c | out |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

Sum of minterms yields

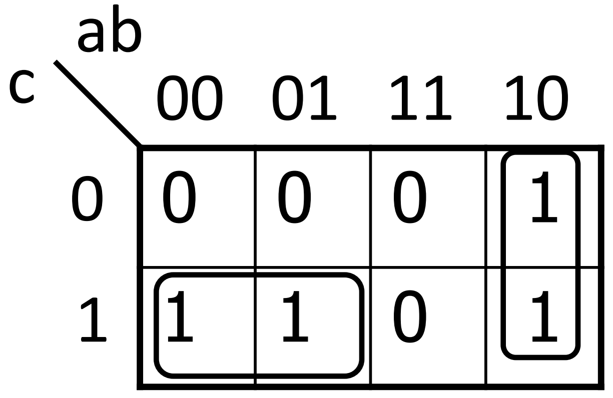
out =

$$\overline{a}\,\overline{b}c + \overline{a}bc + a\overline{b}\,\overline{c} + a\overline{b}c$$

| c \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 |

K-maps identify which inputs are relevant to the output

# Minimization with K-Maps



(1) Circle the 1's (see below)

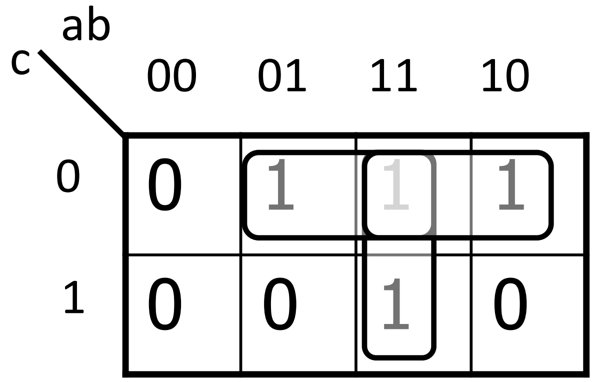
(2) Each circle is a logical component of the final equation

$= a\bar{b} + \bar{a}c$

**Rules:**

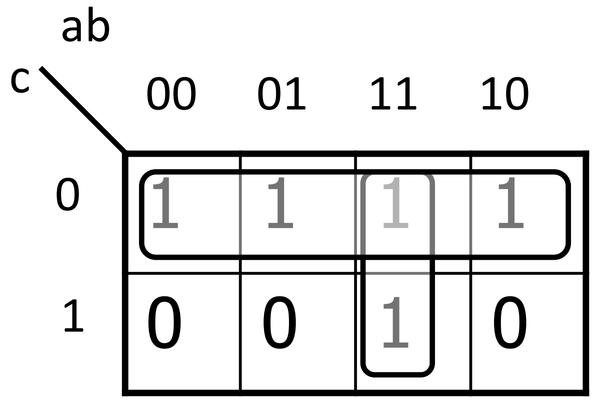
- Use fewest circles necessary to cover all 1's
- Circles must cover *only* 1's
- Circles span rectangles of size power of 2 (1, 2, 4, 8…)
- Circles should be as large as possible (all circles of 1?)
- Circles may wrap around edges of K-Map
- 1 may be circled multiple times *if* that means fewer circles

# Karnaugh Minimization Tricks (1)



Minterms can overlap

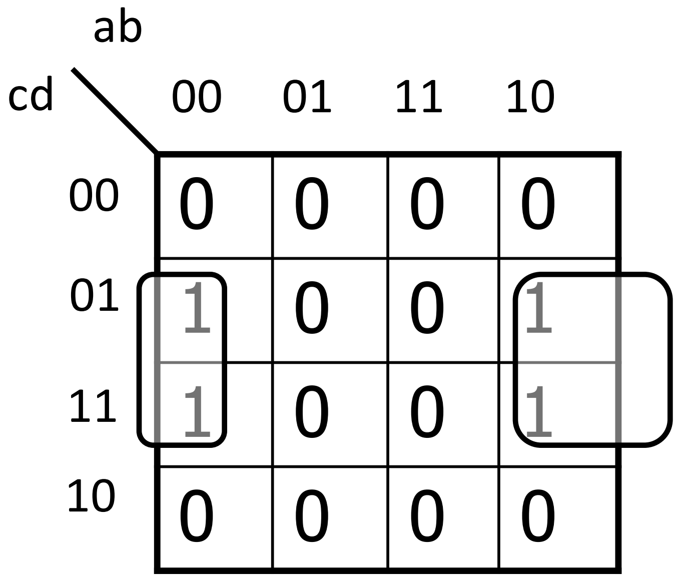
out = $b\overline{c} + a\overline{c} + ab$



Minterms can span 2, 4, 8 or more cells

out = $\overline{c} + ab$

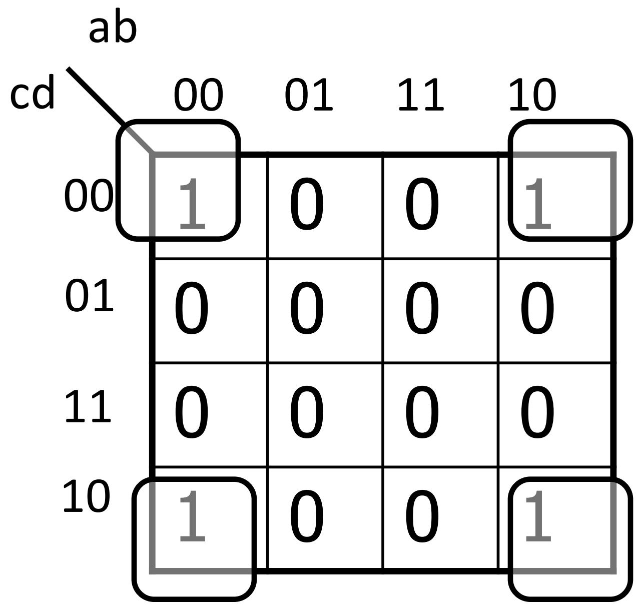
# Karnaugh Minimization Tricks (2)



| cd \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 |

The map wraps around

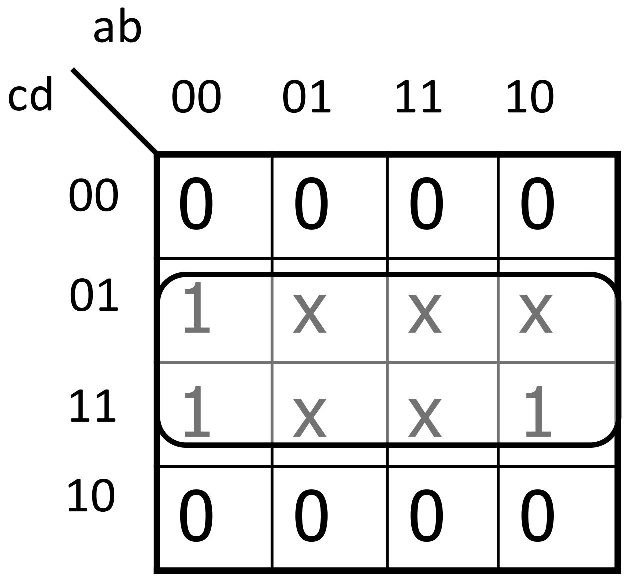
out = $\overline{b}d$



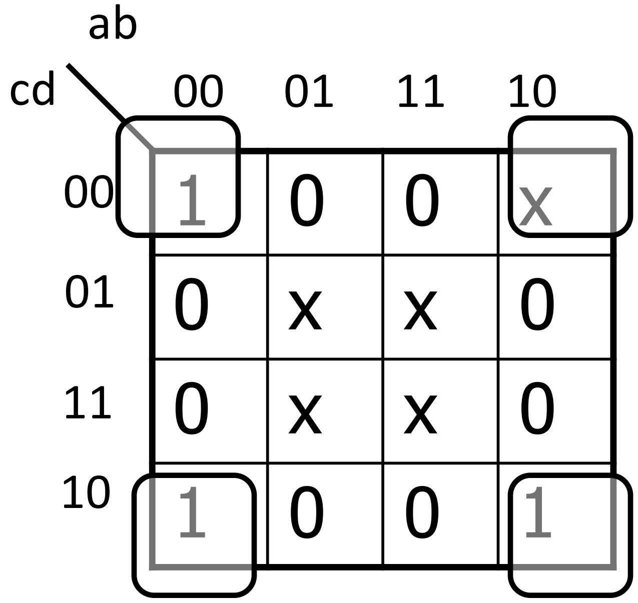
| cd \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 1 |

out = $\overline{b}\,\overline{d}$

# Don’t Cares



| cd \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | x | x | x |
| 11 | 1 | x | x | 1 |
| 10 | 0 | 0 | 0 | 0 |



| cd \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | x |
| 01 | 0 | x | x | 0 |
| 11 | 0 | x | x | 0 |
| 10 | 1 | 0 | 0 | 1 |

“Don’t care” values can be interpreted individually in whatever way is convenient

- assume all x’s = 1

→ out = d

- assume middle x’s = 0 (ignore them)
- assume 4th column x = 1

→ out = $\bar{b}\ \bar{d}$

## Takeaway

- Binary —two symbols: true and false—is the basis of Logic Design
- More than one Logic Circuit can implement same Logic function. Use Algebra (Identities) or Truth Tables to show equivalence.
- Any logic function can be implemented as “sum of products”. Karnaugh Maps minimize number of gates.

## Summary

Most modern devices made of billions of transistors

- You will build a processor in this course!
- Modern transistors made from semiconductor materials
- Transistors used to make logic gates and logic circuits

We can now implement any logic circuit

- Use P- & N-transistors to implement NAND/NOR gates
- Use NAND or NOR gates to implement the logic circuit
- *Efficiently*: use K-maps to find required minimal terms