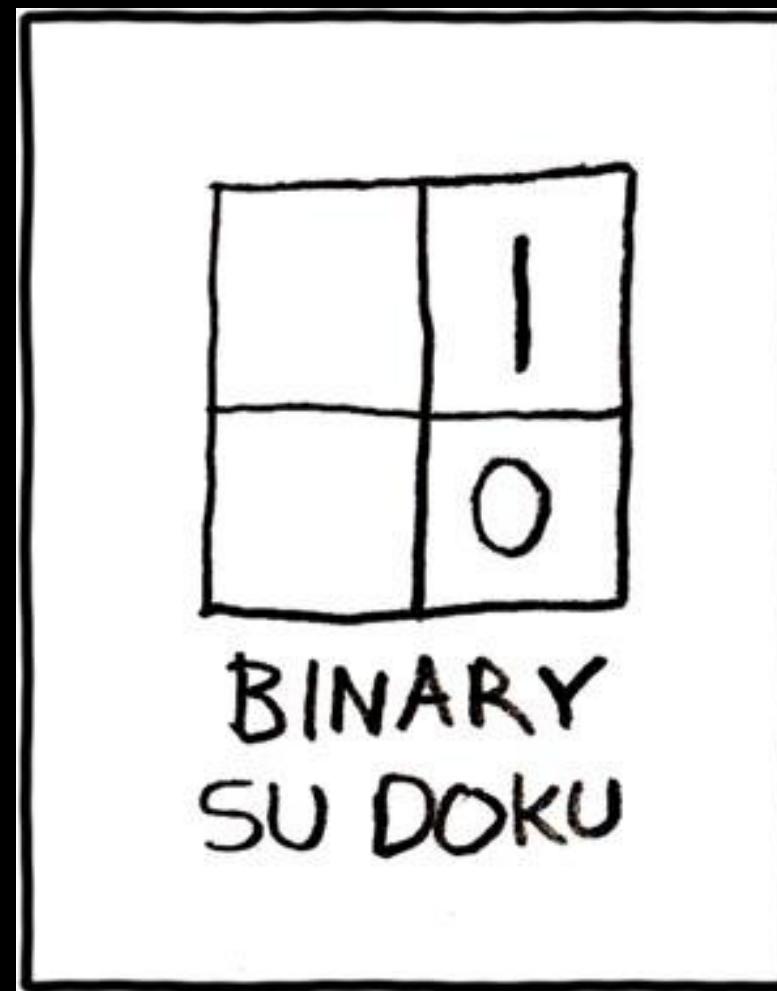


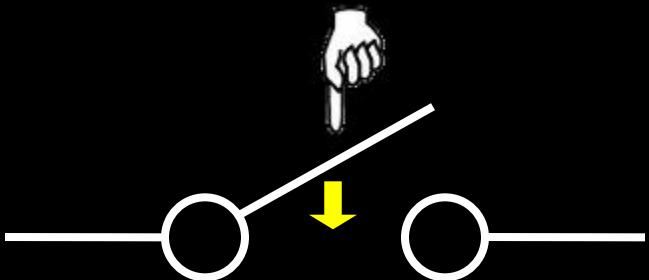
Gates and Logic



See: P&H Appendix C.2, C.3

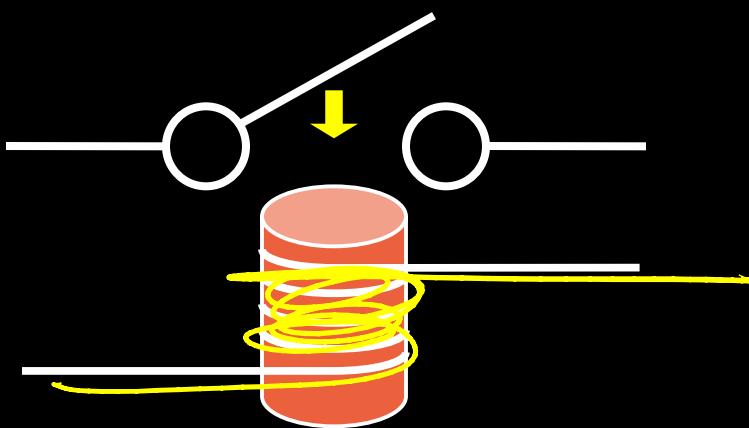


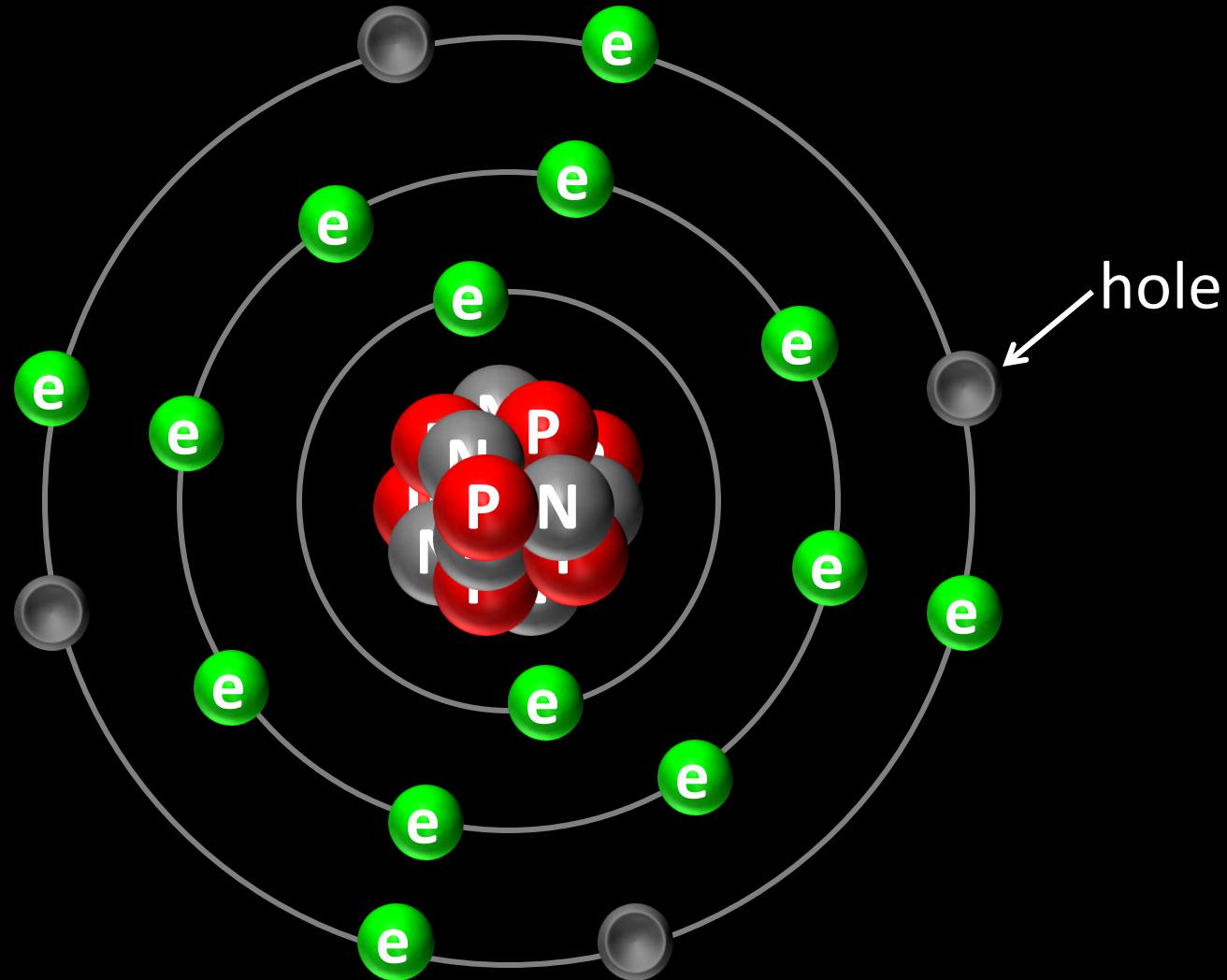
- Acts as a *conductor* or *insulator*
- Can be used to build amazing things...

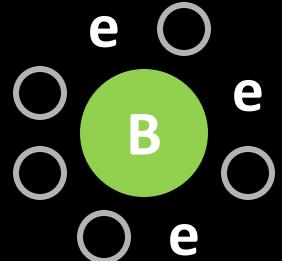




- One current controls another (larger) current
- Static Power:
 - Keeps consuming power when in the *ON* state
- Dynamic Power:
 - Jump in power consumption when switching



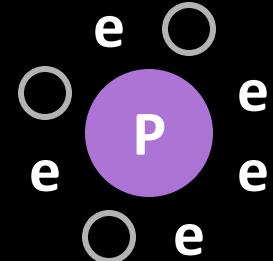




Boron

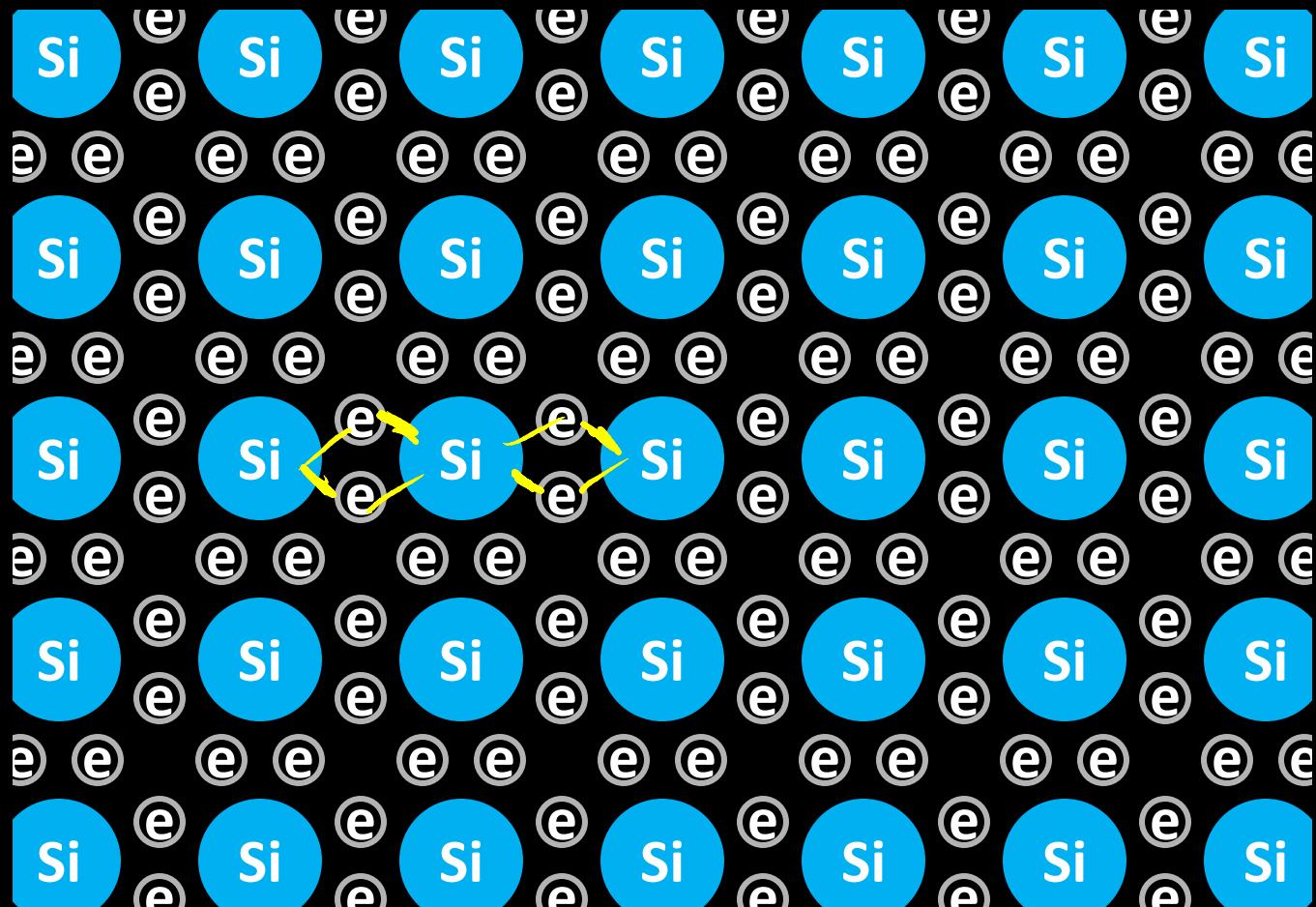


Silicon

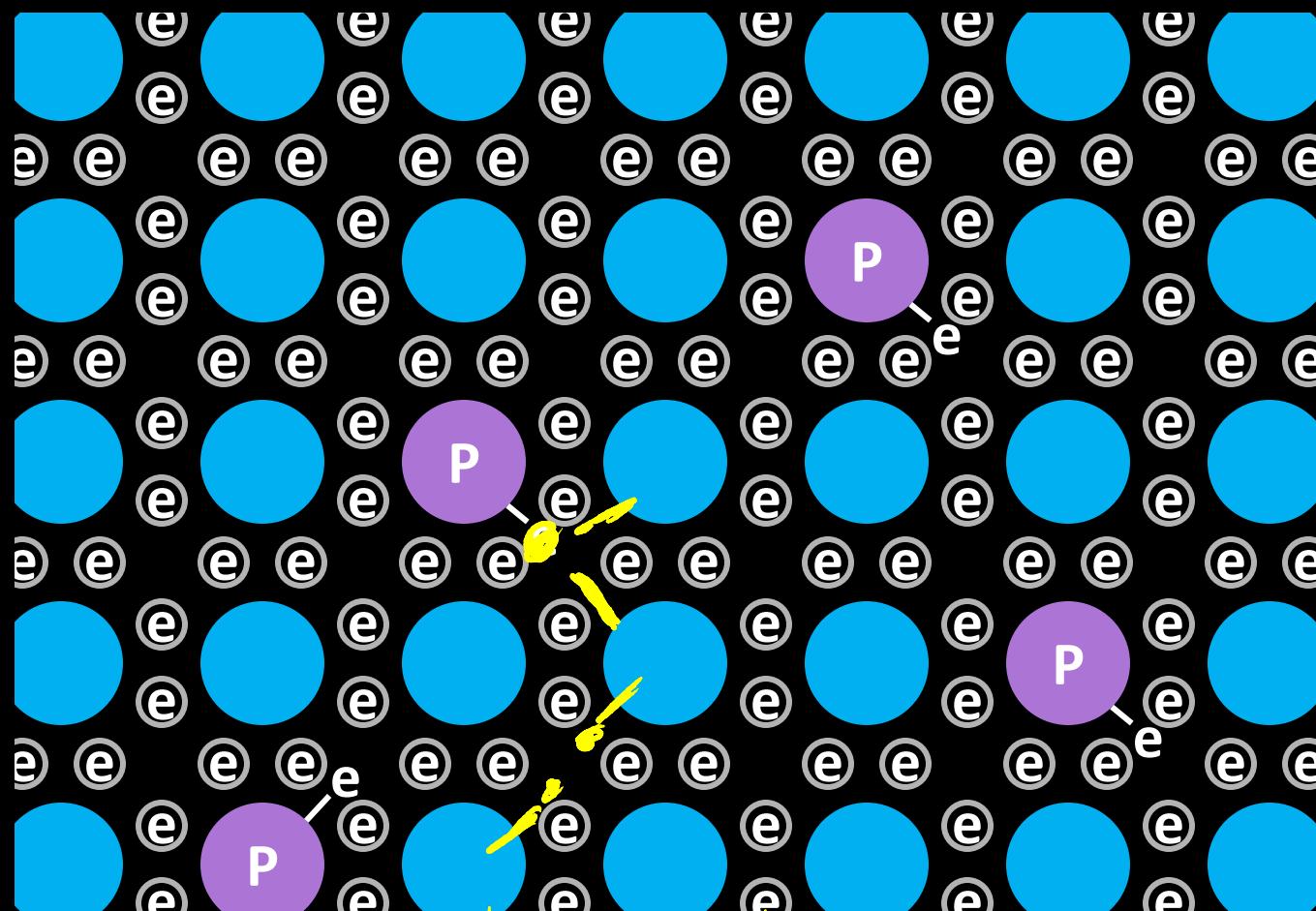


Phosphorus

Silicon

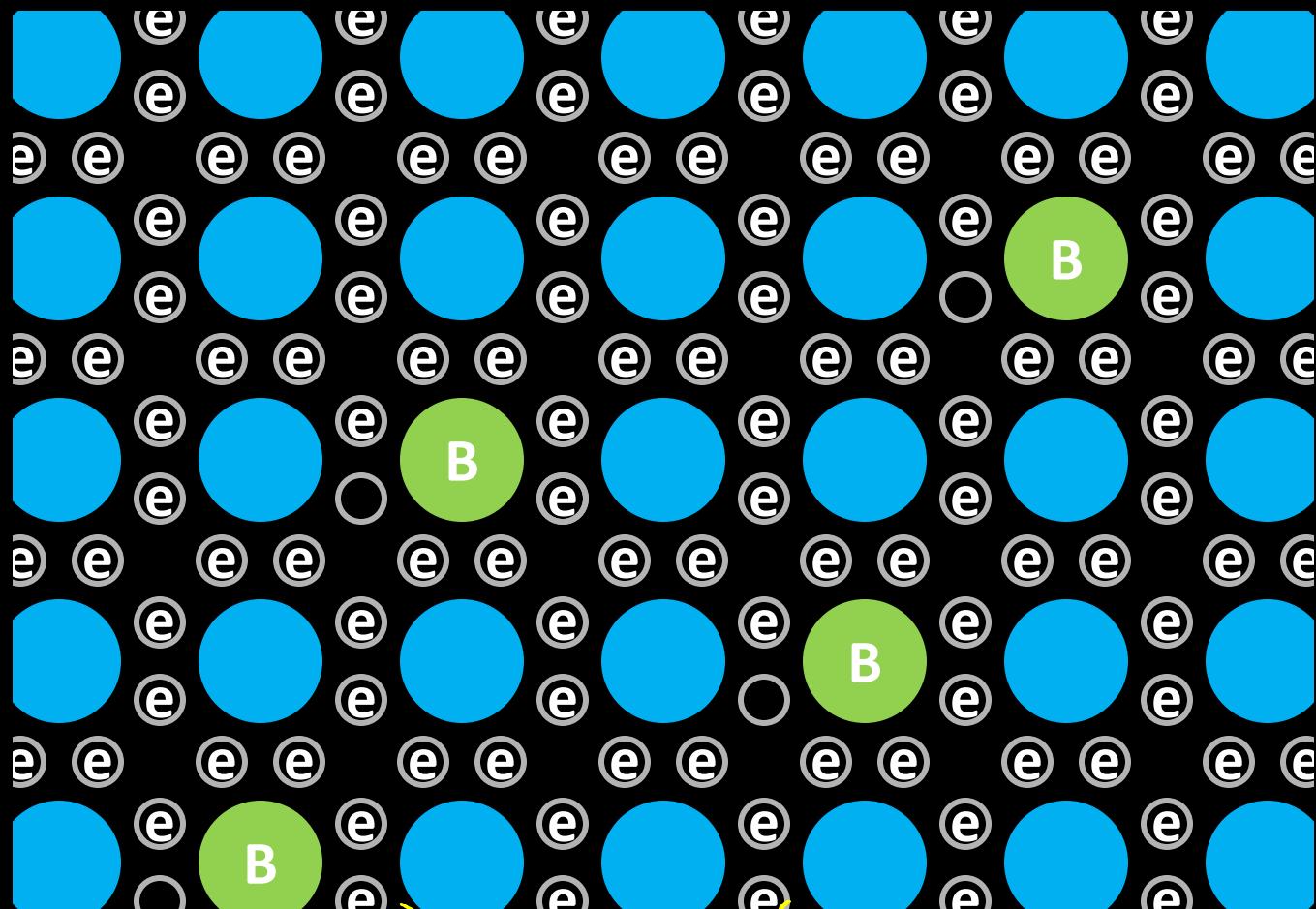


N-Type: Silicon + Phosphorus



mobile electrons \Rightarrow conductor
 $h^- + v^- \rightarrow$ depleted = ins.

P-Type: Silicon + Boron



mobile holes \Rightarrow cond.
= \Rightarrow depleted \Rightarrow ins.



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Insulator



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p-type (Si+Boron)
has mobile holes:

low voltage (depleted)
→ insulator

high voltage (mobile holes)
→ conductor

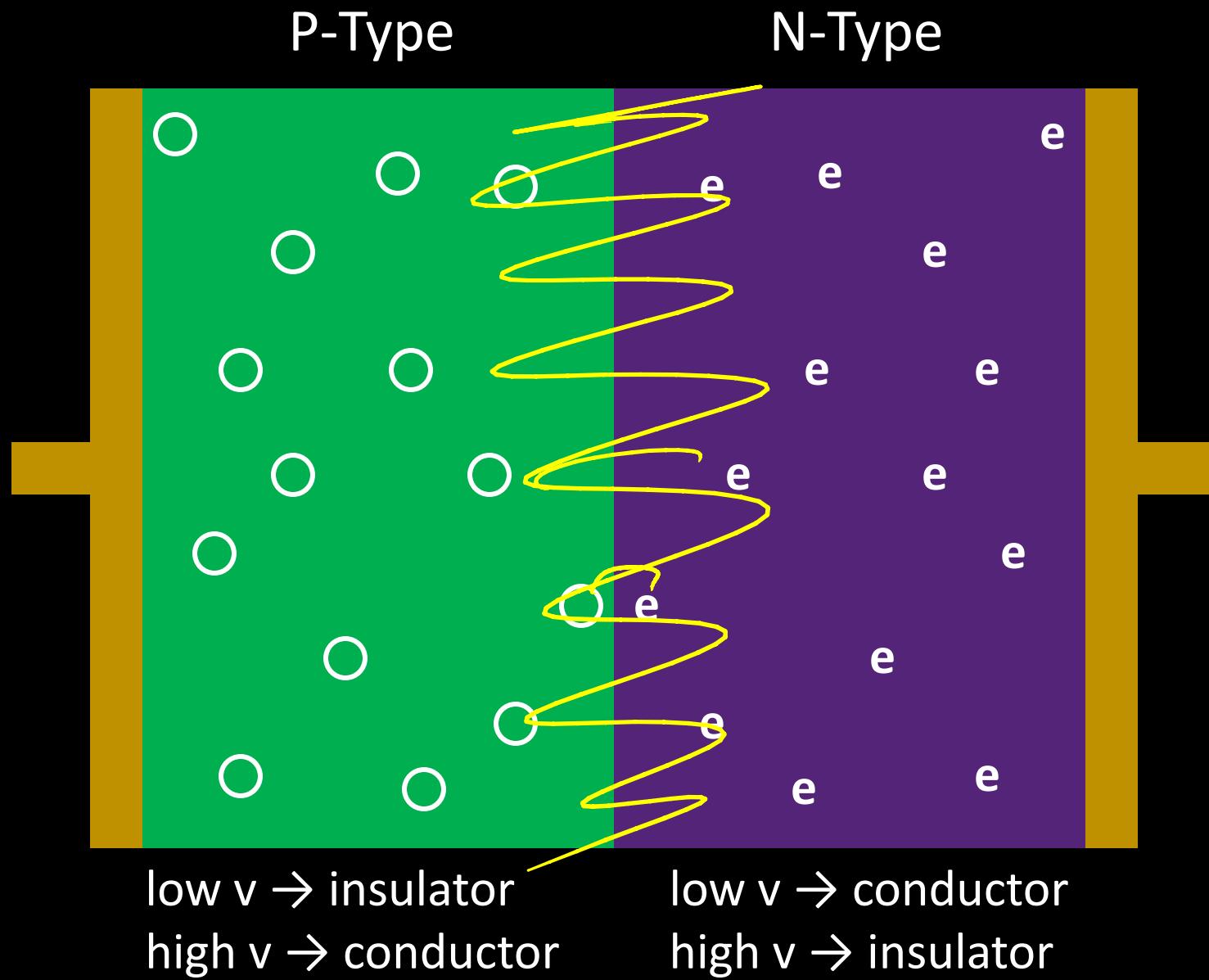


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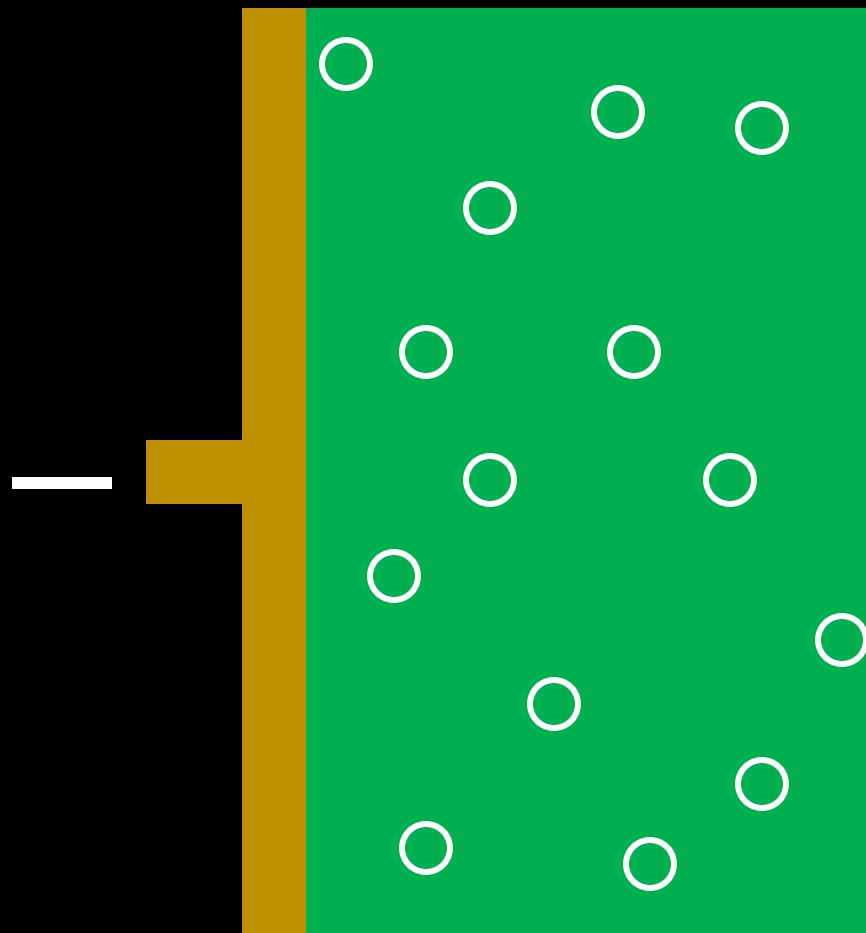
n-type (Si+Phosphorus)
has mobile electrons:

low voltage (mobile electrons)
→ conductor

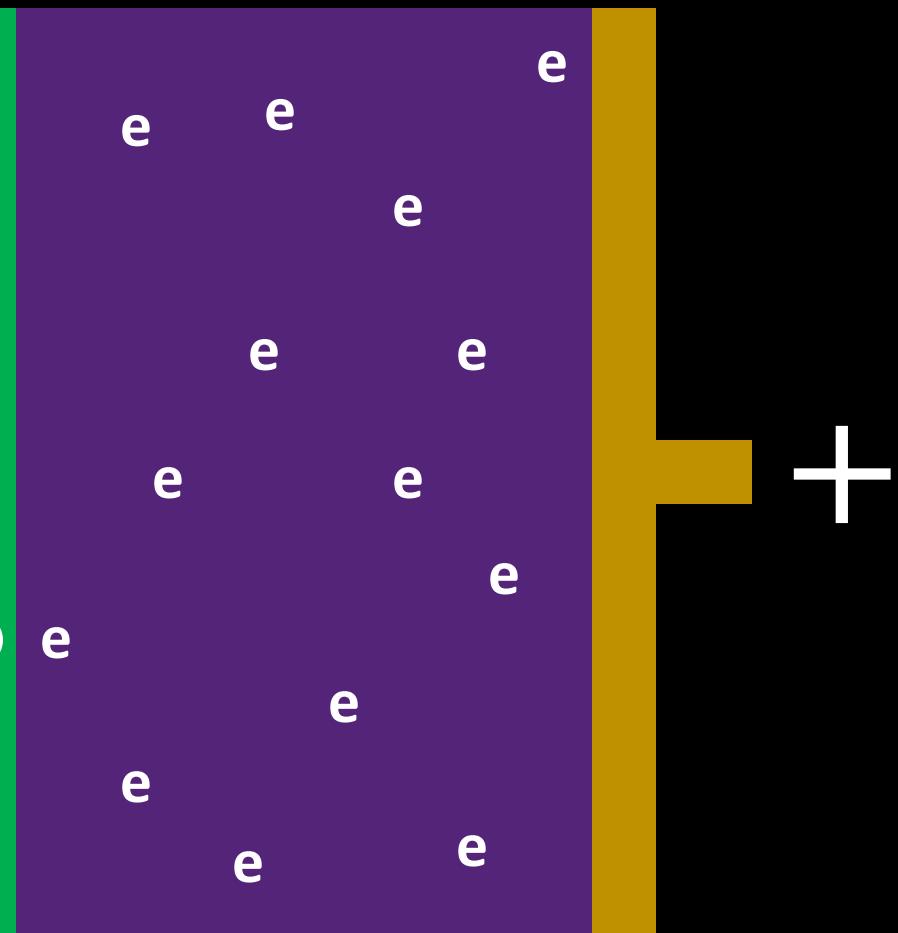
high voltage (depleted)
→ insulator



P-Type

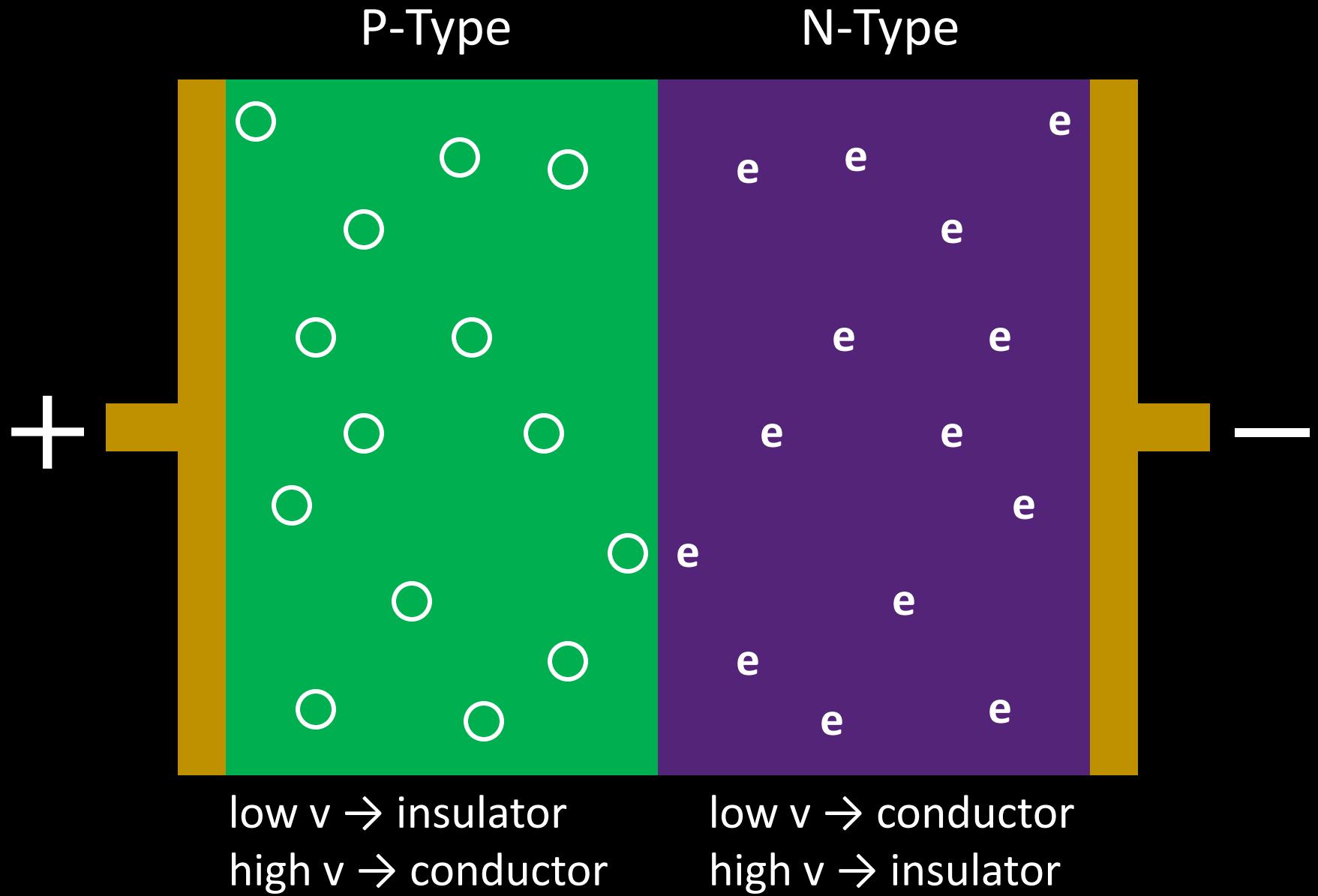


N-Type

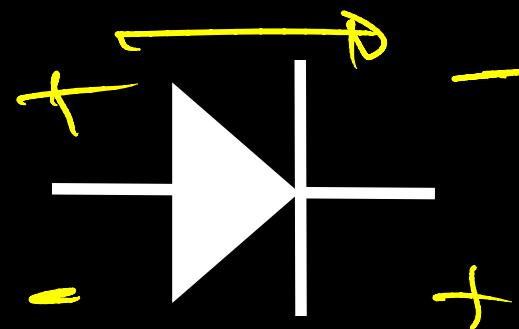
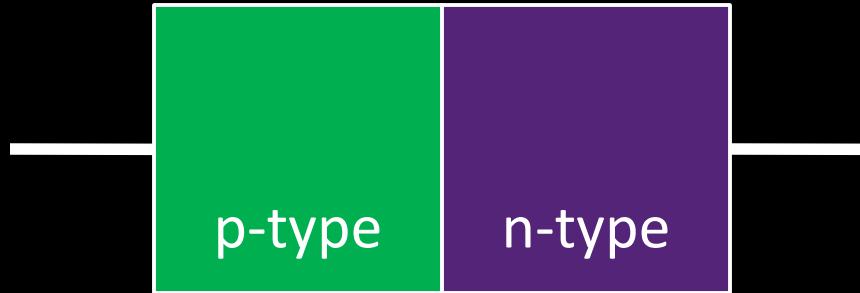


low $v \rightarrow$ insulator
high $v \rightarrow$ conductor

low $v \rightarrow$ conductor
high $v \rightarrow$ insulator



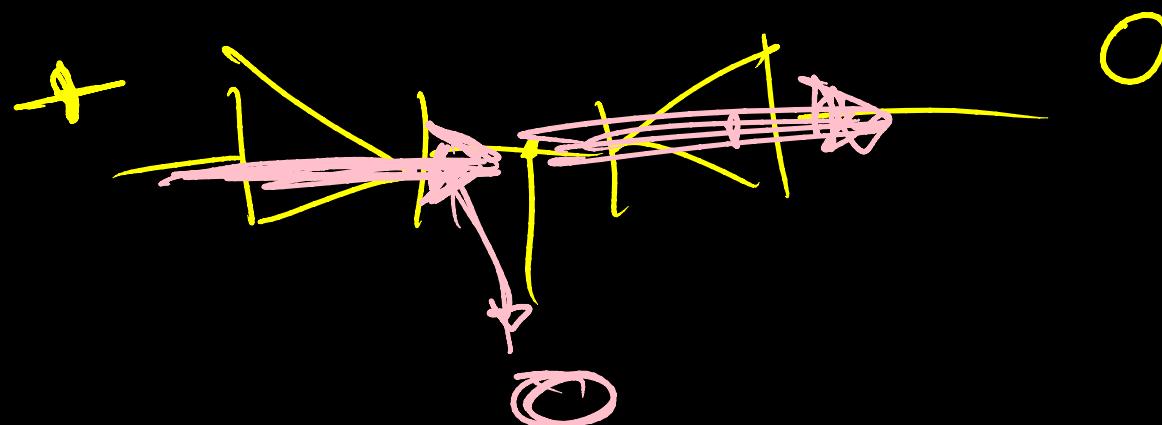
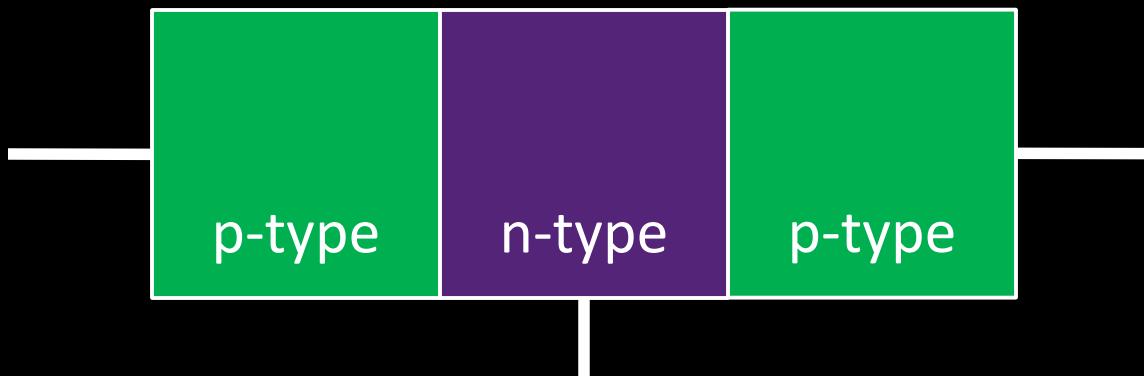
PN Junction “Diode”

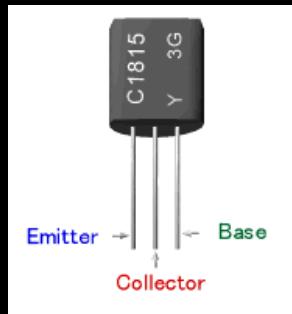


Conventions:

$v_{dd} = v_{cc} = +1.2v = +5v = hi$

$v_{ss} = v_{ee} = 0v = gnd$

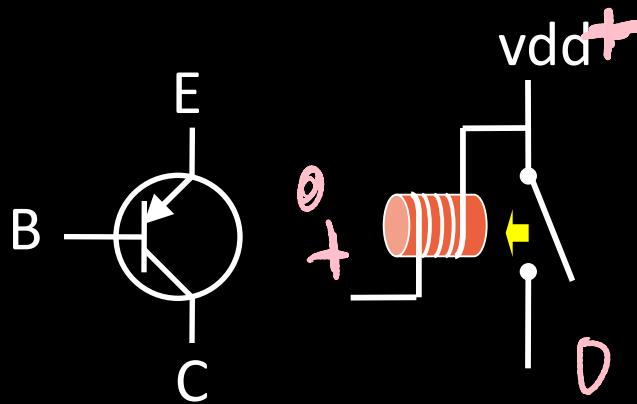
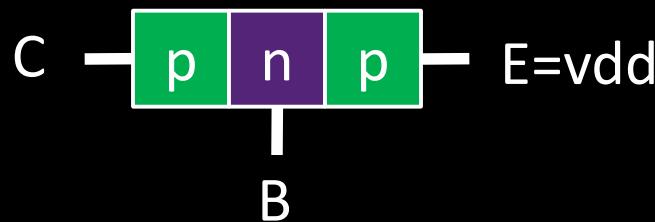




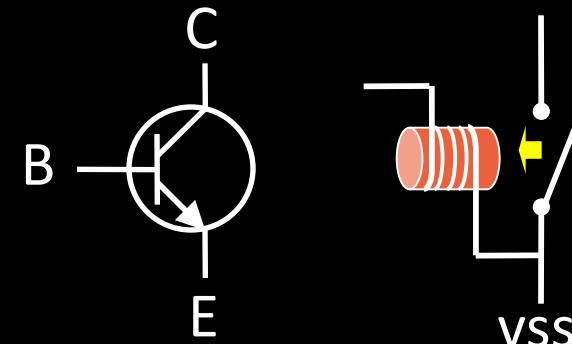
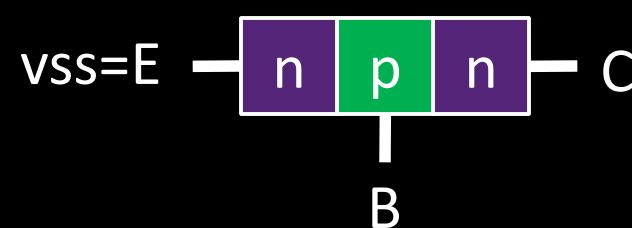
- Solid-state switch: The most amazing invention of the 1900s

Emitter = “input”, Base = “switch”, Collector = “output”

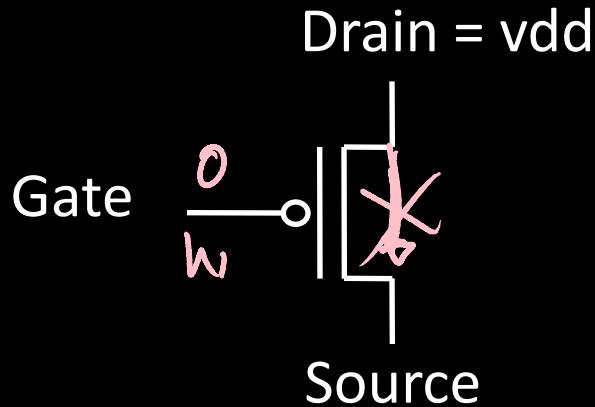
PNP Transistor



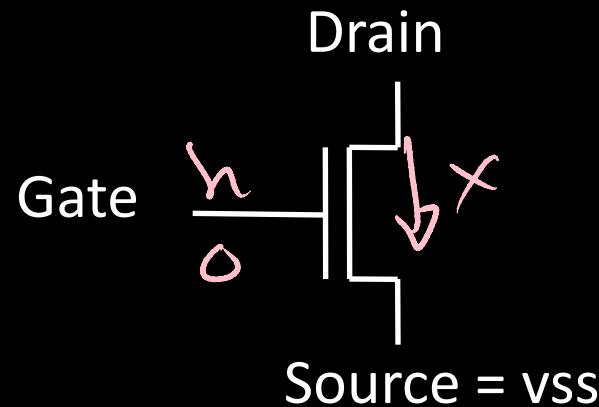
NPN Transistor



P-type FET

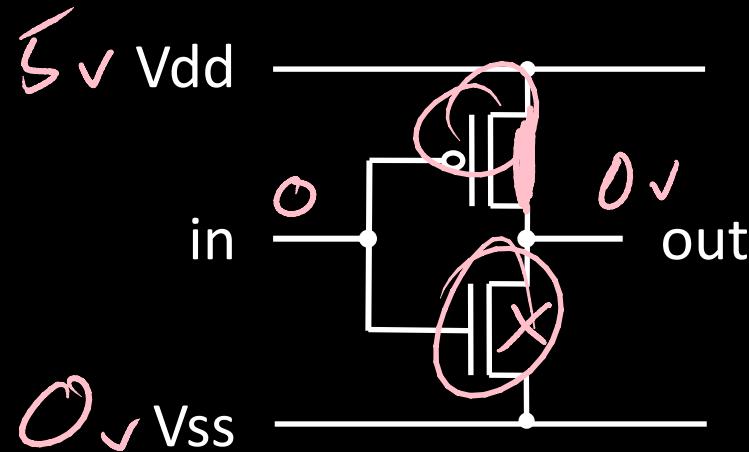


N-type FET



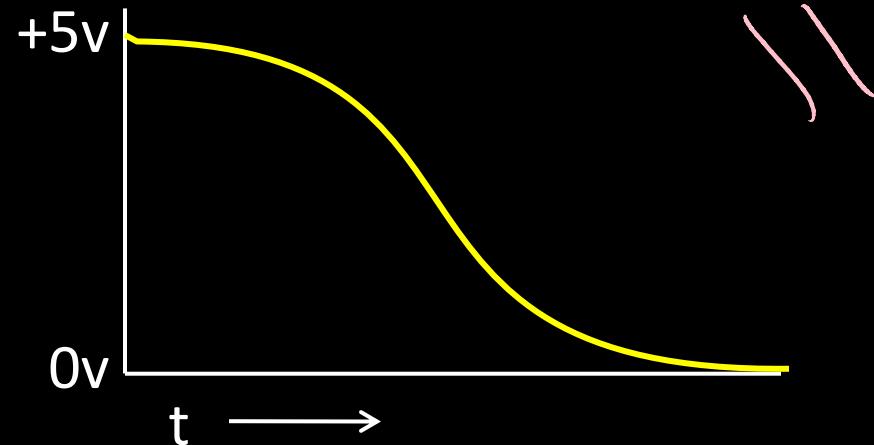
- Connect Source to Drain when Gate = l_0
 - Drain must be v_{dd} , or connected to source of another P-type transistor

- Connect Source to Drain when Gate = hi
 - Source must be vss, or connected to drain of another N-type transistor



In	Out
✓	✗
✗	✓

voltage



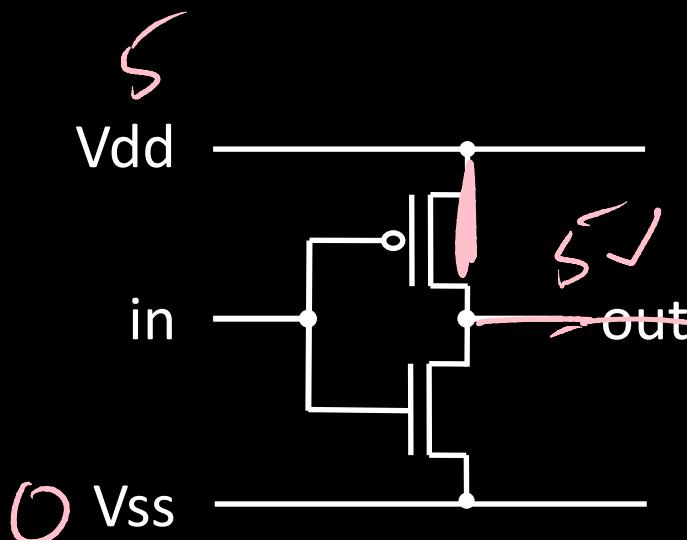
Gate delay

- transistor switching time
- voltage, propagation, fanout, temperature, ...

CMOS design

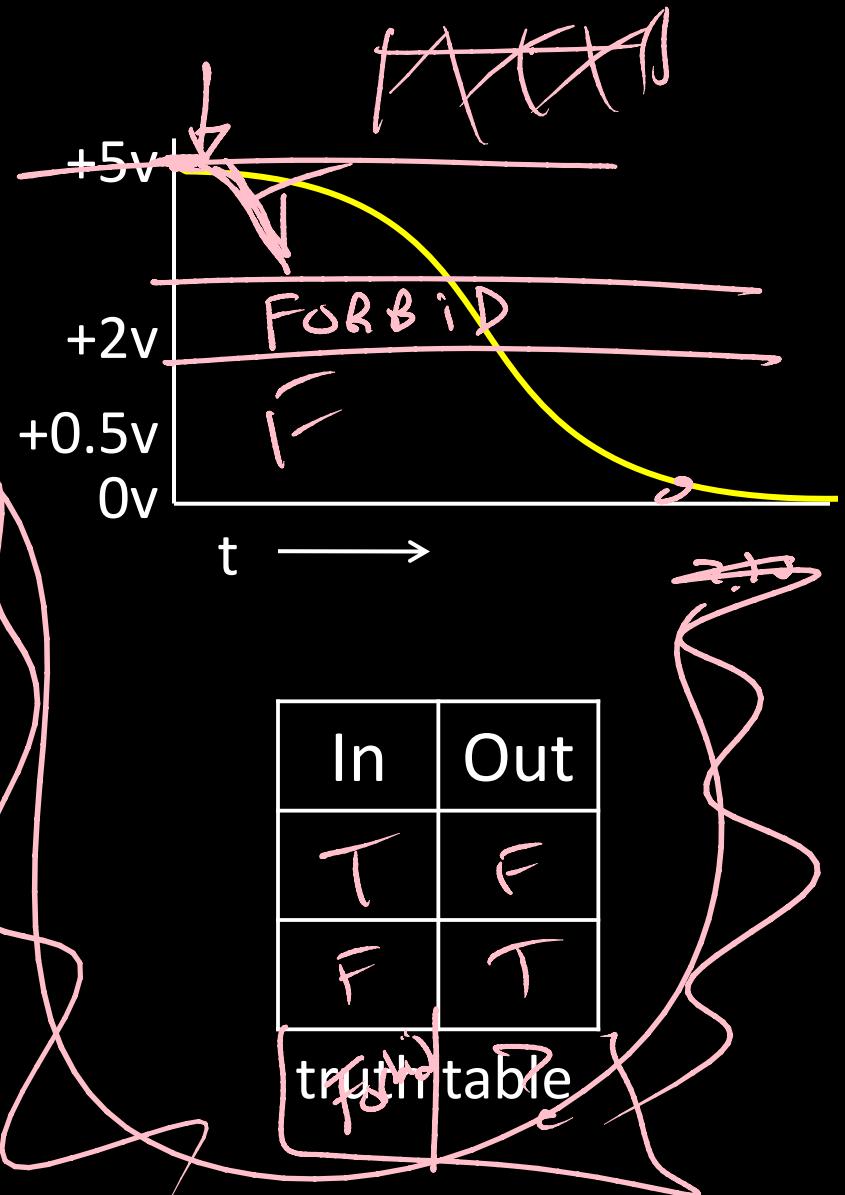
(complementary-symmetry metal–oxide–semiconductor)

- Power consumption = dynamic + leakage



In	Out
+5v	0v
0v	+5v

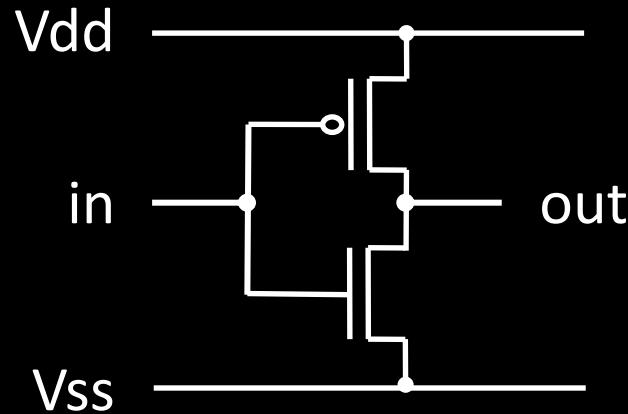
voltage



Conventions:

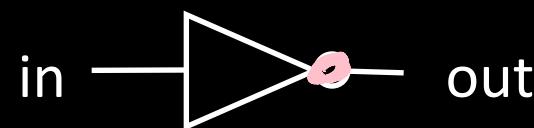
vdd = vcc = +1.2v = +5v = hi = true = 1

vss = vee = 0v = gnd = false = 0



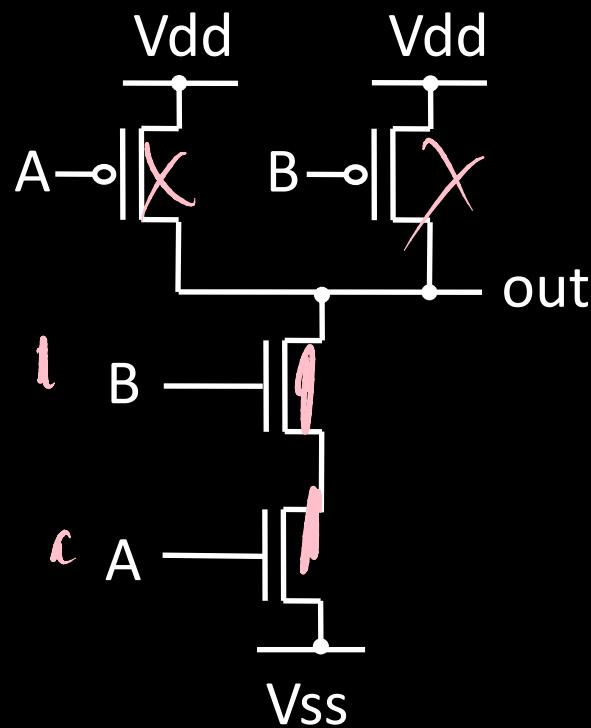
Function: NOT

- Symbol:



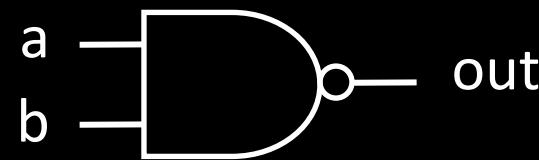
In	Out
0	1
1	0

Truth table

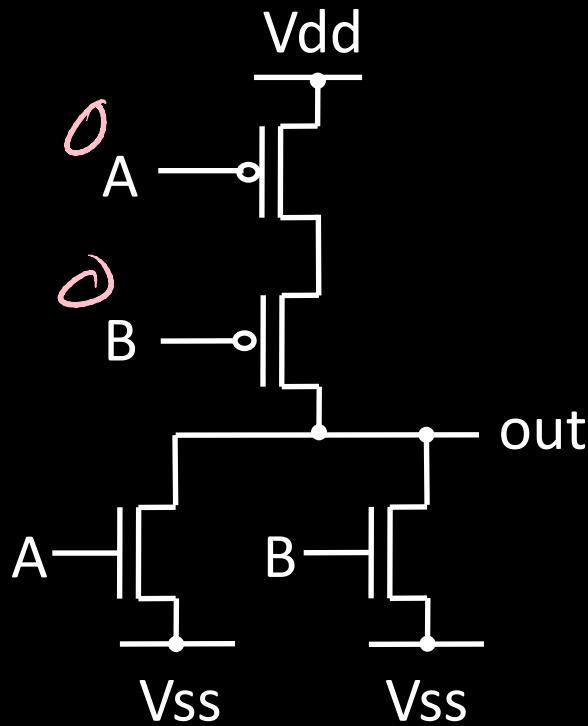


Function: NAND

- Symbol:

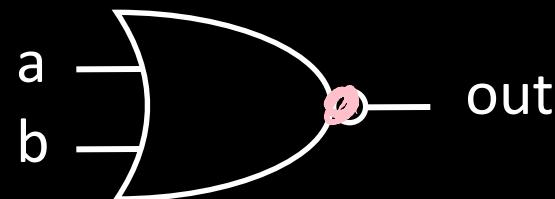


A	B	out
0	0	1
0	1	1
1	0	1
1	1	0



Function: NOR

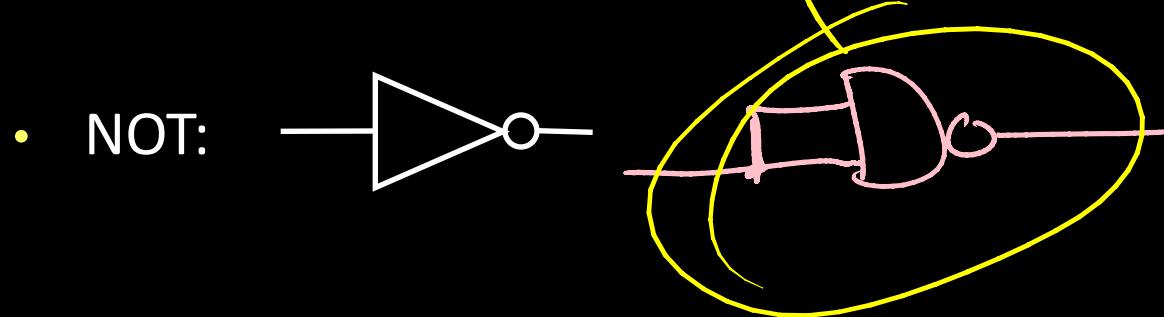
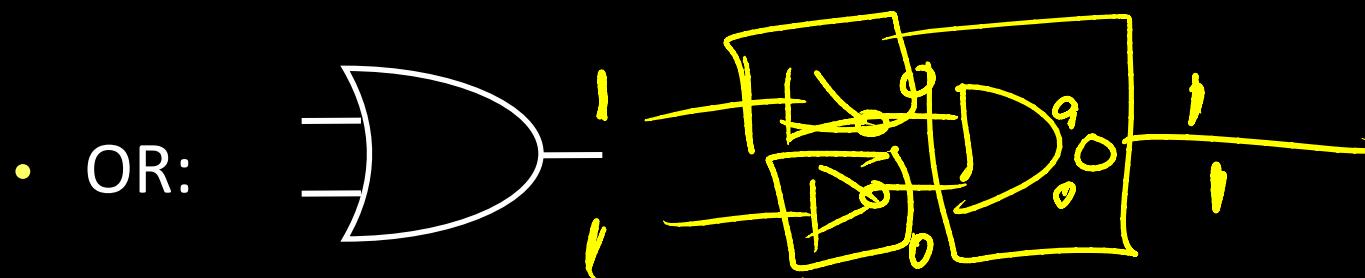
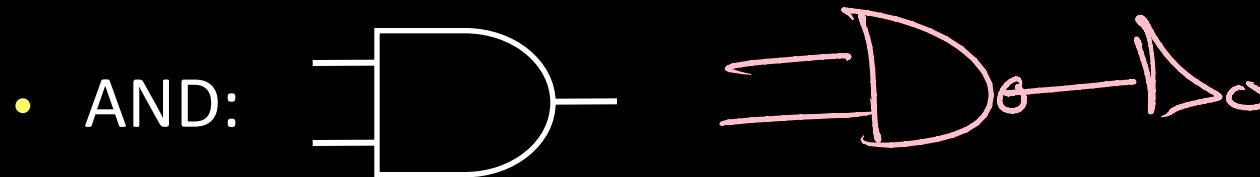
- Symbol:



A truth table for a NOR gate with three columns: A, B, and out. The rows show all combinations of A and B. The output 'out' is 1 when both A and B are 0, and 0 for all other input combinations. A pink oval highlights the row where A and B are both 0.

A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

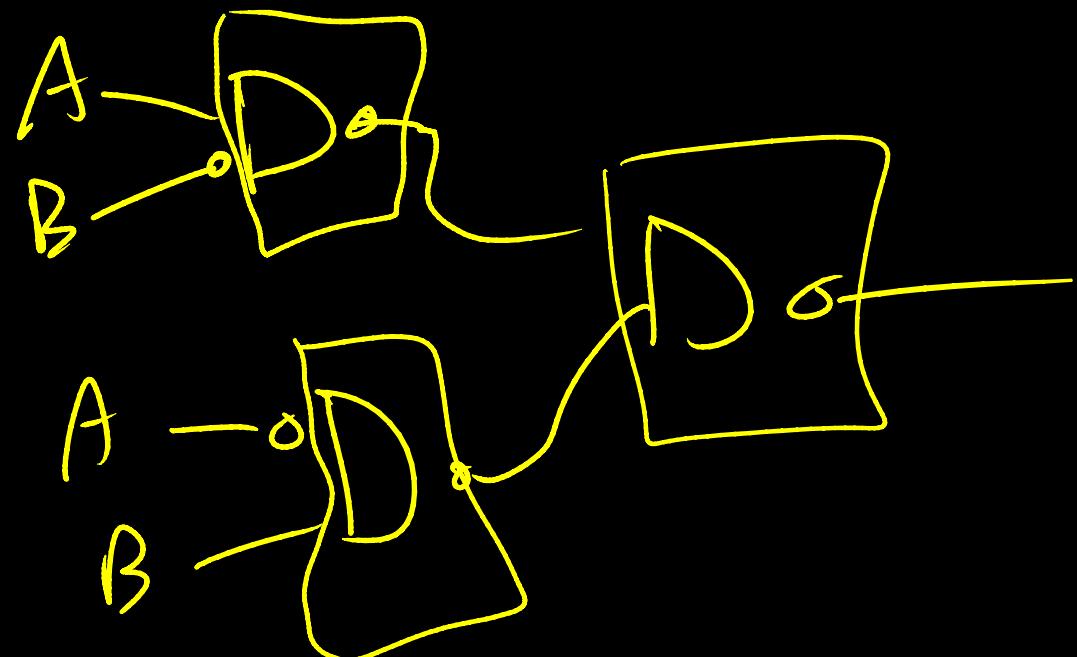
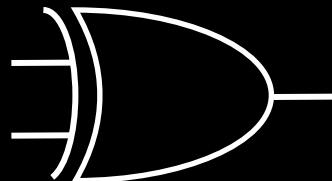
$$\boxed{\text{NOT } (\text{X and Y}) = \text{NOT X} \text{ or } \text{NOT Y}}$$



NAND is universal (so is NOR)

- Can implement any function with just NAND gates
 - De Morgan's laws are helpful (pushing bubbles)
- useful for manufacturing

E.g.: XOR (A, B) = A or B but not both ("exclusive or")



Proof: ?

Some notation:

- constants: true = 1, false = 0
- variables: a, b, out, ...
- operators:

$\text{AND}(a, b)$	$= a \cdot b$	$= a \& b$	$= a \wedge b$
$\text{OR}(a, b)$	$= a + b$	$= a b$	$= a \vee b$
$\text{NOT}(a)$	$= \bar{a}$	$= !a$	$= \neg a$

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

$$1 \cdot 0 = 0$$

$$1 \cdot 1 = 1$$

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$$a + 0 = a$$

$$a + 1 = 1$$

$$a + \bar{a} = 1$$

$$a \cdot 0 = 0$$

$$a \cdot 1 = a$$

$$a \cdot \bar{a} = 0$$

$$\overline{(a + b)} = \bar{a} \bar{b}$$

$$\overline{(a \cdot b)} = \bar{a} + \bar{b}$$

$$a + a \cdot b = a$$

$$a(b+c) = ab + ac$$

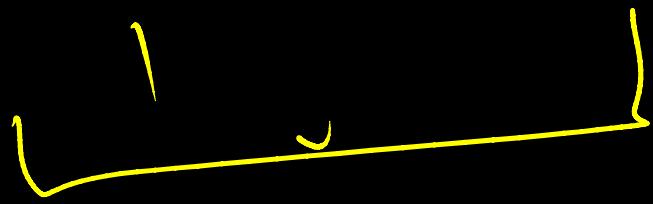
$$\overline{a(b+c)} = \bar{a} + \bar{b}\bar{c}$$

- functions: gates \leftrightarrow truth tables \leftrightarrow equations
- Example: $(a+b)(a+c) = a + bc$

a	b	c	a+b	a+c	LHS	bc	RHS
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

$$(a+b)(a+c)$$

$$\frac{a \cdot a + a \cdot c + b \cdot a + b \cdot c}{a \cdot 1} = a(1 + c + b) + b \cdot c$$



$$\underbrace{a \cdot 1}_a + b c + b c$$