

CS 3220: PRELIM 2 EXAMPLE QUESTIONS

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WHATS IN THIS DOCUMENT

These questions are intended to be somewhat representative of the type of questions that could arise on the second prelim. They are focused on topics that have not been covered by HW problems.

A ROUGH OUTLINE OF THE TOPICS:

A roughly comprehensive list of topics that may be covered by the second prelim is (two good resources are the introductory probability and statistics books given on the course website):

- Continuous random variables
- Tail bounds
- limit theorems
- Multivariate Normal distributions
- Sampling random variables (rejection sampling and inverse CDF methods)
- Maximum likelihood estimation
- Sample mean and sample variance
- Unbiased and consistent estimators
- Sample covariance estimation
- Covariance matrix estimation
- Principle component analysis

QUESTION 1:

Say you are given independent and identically distributed samples X_1, X_2, \dots, X_n from a uniform distribution on $[0, \theta]$ and θ is unknown. What is the MLE for θ ?

QUESTION 2:

Consider a sequence of independent and identically distributed random vectors X_1, X_2, \dots, X_n where $X_i \in \mathbb{R}^d$ and $n \geq d$. Let $\bar{\mu}_\ell$ be the sample mean of $(X_i)_\ell$ and $\bar{\sigma}_\ell^2$ be the sample variance of $(X_i)_\ell$. Show that if any two entries of the vectors X_i are perfectly correlated, *i.e.*

$$\frac{\sum_{i=1}^n [(X_i)_\ell - \bar{\mu}_\ell][(X_i)_k - \bar{\mu}_k]}{\bar{\sigma}_k \bar{\sigma}_\ell} = 1$$

for some $k \neq \ell$, then $\hat{\Sigma} = \hat{X}\hat{X}^T$ has rank $< d$.