CS 3220: PRELIM 1 EXAMPLE QUESTIONS
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WHAT'S IN THIS DOCUMENT
These questions are intended to be somewhat representative of the type of questions that could arise on the first prelim.

QUESTION 1
Here, we consider some properties of $\|A\|_F$. For what follows $A \in \mathbb{R}^{n \times n}$

(a) Prove that

$$\|A\|_F^2 = \sum_{i=1}^{n} \|A(:, i)\|_2^2.$$ 

(b) For any two orthogonal matrices $Q_1 \in \mathbb{R}^{n \times n}$ and $Q_2 \in \mathbb{R}^{n \times n}$ prove that

$$\|Q_1AQ_2\|_F = \|A\|_F$$

(c) Prove that

$$\|A\|_F = \sqrt{\sum_{i=1}^{n} \sigma_i^2},$$

where $\sigma_1, \ldots, \sigma_n$ are the singular values of $A$.

QUESTION 2
Here, we ask you to interpret the condition number of a $2 \times 2$ matrix geometrically. (Hint: pictures are useful here!)

1. We saw that the SVD of a $2 \times 2$ matrix allows us to view any matrix $A$ as mapping a circle to an ellipse. If $A$ becomes increasing ill-conditioned what is geometrically happening to the ellipse?

2. Geometrically argue why for an ill-conditioned matrix a small relative change in $b$ can result in a big relative change in $x$.

QUESTION 3
Say you are given a symmetric matrix $A$ and tasked with computing the algebraically smallest eigenvalue. Using only the power method (applied to $A$ or matrices related to $A$), how might you go about doing this? (Hint: think about how the eigenvalues/vectors of $A - \gamma I$ relate to those of $A$ for any scalar $\gamma \in \mathbb{R}$.)