Problem 1: LU factorization, upside-down and backwards.

The Matlab `lu` function computes a factorization $PA = LU$ from a square matrix $A$, with $P$ a permutation, $L$ a unit lower triangular matrix (unit meaning it has ones on the diagonal) and $U$ an upper triangular matrix. Several things about this factorization are merely arbitrary conventions, and the same basic procedure with minor changes could be used to compute other factorizations. For each of the following factorizations modify the given LU code to produce nice clean code to compute the factors directly, and then also provide a few-line (at most 5) Matlab expression to compute the same result by calling the built in `lu()` routine. Please fill in your code in `luShuffle.m`. Also, please print out and hand in the output from the test script.

(a) $AP = LU$ (with 1s on the diagonal of $U$)
(b) $PA = UL$ (with 1s on the diagonal of $U$)
(c) $PA = LU$ (with 1s on the diagonal of $U$)
(d) $PA = LDU$ (with 1s on the diagonal of $L$ and $U$, and $D$ is a diagonal matrix);

Each of these can be achieved using the Gaussian elimination algorithm, but rearranging the order in which rows and columns are processed.

Hints:

1. We will place special emphasis on conciseness, elegance, and vectorization.
2. Make sure you have a good understanding of how the code we have provided works before doing anything.
3. Most of the changes you will need to make to the code provided are indexing changes. The structure to the code should remain very similar. If you need to add a line of code or two, make sure you really need them.
4. As we discussed in lecture, the code is set up to represent permutations as vectors of integers, rather than as matrices. Where one would write $PA$ on
paper, one then writes $A(p,:)$ in code, and similarly $AP$ is $A(:,p)$. We will also express diagonal matrices as a vector (though it’s OK to multiply by a diagonal matrix using \texttt{diag(d)*A}; behind the scenes Matlab does the right thing).

Also complete Review Questions 2.3, 2.10, 2.21, 2.50, and 2.51.