CS 322 Homework 7

out: Thursday 12 April 2007
due: Wednesday 18 April 2007

Ordinary Differential Equations

Problem 1: Problem 7.1 in Moler

Problem 2: Stiff Systems

Take the sample ODE given on Moler p. 205

$$
\begin{aligned}
\dot{y} &= y^2 - y^3 \\
y(0) &= \delta \\
0 \leq t &\leq \frac{2}{\delta}
\end{aligned}
$$

1. Implement three MATLAB functions

   ```
   function y = intFlameForwardEuler(delta, h)
   function y = intFlameMidpoint(delta, h)
   function y = intFlameBackEuler(delta, h)
   ```

   that integrate the above ODE given the parameter δ and the fixed timestep h to step over the specified interval of t using the appropriate fixed-stepsize integrator (Forward Euler, Midpoint, and Backward Euler). Your functions should return a vector $\mathbf{y}$ where $\mathbf{y}(i)$ is the computed value of y at timestep $t_i = ih$.

2. Plot the computed value of y for $\delta = 0.0005$ with each of your methods using at least three different choices of timesteps. What do you observe about the solution computed by each of these methods? What can you say about the order of accuracy (how fast the answer gets better as a function of stepsize h) and stability (stepsizes h for which the answer does not diverge) for each of the three methods on this problem? Submit your plots along with your answer. Your code should be submitted through CMS.

Problem 3: [$\star$] Error analysis

Consider the following ODE integrator, which comes from the Adams family of methods

$$u_{n+1} = u_n + \frac{h}{2}\left(3f(t_n, u_n) - f(t_{n-1}, u_{n-1})\right)$$

Using the methods discussed on Moler p. 215, show that the local discretization error of this method is $O(h^3)$, and so the method is of order 2.