CS 322 Homework 4

out: Thursday 1 March 2007
due: Wednesday 7 March 2007

Problem 1: QR mechanics

1 Prove that a Householder reflection is symmetric and orthogonal.

2 Show that the Householder vector given on p. 147 in Moler produces a transformation that zeros out all but the k^{th} element of x.

⋆3 Explain why Moler's recommendation for choosing the sign of σ improves accuracy.

4 How many floating-point operations (flops) are required to multiply an $m \times n$ matrix by a Householder reflection?

5 What size matrices are updated by the $n-1$ steps of a QR factorization of an m by n matrix?

6 If the Q factor is computed as part of the factorization, What size matrices are updated by the $n-1$ steps of that process? What factor of additional work does this add?

7 Time the Matlab operations "`qr(A);`" and "`[Q,R] = qr(A);`" using `tic` and `toc`, for matrices large enough to take a few seconds to factor. Is your answer to (6) borne out? Explain.

Problem 2: Subspaces

Find the subspaces $\mathrm{ran}(A)$, $\mathrm{ran}(A)^\perp$, $\mathrm{null}(A)$, and $\mathrm{null}(A)^\perp$ for each of the following full-rank matrices. Give orthonormal bases for each subspace and explain how you computed them in your head or by using the QR factorization. Simpler bases found by reasoning are preferable to messier ones found by computation.

1. $\begin{bmatrix} 3 & 3 & 8 \\ 2 & 8 & 3 \\ 7 & 2 & 3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 6 & 7 \\ 8 & 5 & 4 \\ 2 & 3 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 6 & 9 \\ 6 & 7 & 4 \\ 9 & 5 & 7 \\ 7 & 8 & 8 \\ 4 & 4 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 6 & 7 & 0 & 0 \\ 8 & 5 & 4 & 0 & 0 \\ 2 & 3 & 9 & 0 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 9 & 1 & 4 & 5 \\ 7 & 6 & 7 & 7 & 1 \\ 3 & 4 & 8 & 2 & 4 \end{bmatrix}$

You don't need to report your answer with more than 4 significant figures.