CS 322 Homework 1: Solutions

out:	Thursday 25 January 2007
due:	**Wednesday 31 January 2007**

Problem 1

The assignment already expressed $\mathbf{M}$ by entry using the dot product operator, i.e. $m_{ij} = \mathbf{a_{i:}} \cdot \mathbf{b_{:j}}$. One way of expressing this in MATLAB is then

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for i = 1:m
    for j = 1:n
        M(i, j) = A(i, :) * B(:, j);
    end
end
```

We note that we can change the order of the loops above without affecting the answer: that is, we could also write it as

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for j = 1:n
    for i = 1:m
        M(i, j) = A(i, :) * B(:, j);
    end
end
```

All three of the methods for Problem 1 share this feature, namely that there are two valid loop orderings for each of the methods. It should be noted that the different loop orderings do not correspond to different methods of solving the problem, though, just different ways of expressing the mathematical equation in code.

As for the other two methods, we can also express each row i of $\mathbf{M}$ as a linear combination of all rows of $\mathbf{B}$, where the coefficients come from the i-th row of $\mathbf{A}$. Mathematically, this equates to

$$\mathbf{m}_{i:} = \sum_{k=1}^{p} a_{ik}\mathbf{b}_{k:}$$

The MATLAB code for this method is then

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for i = 1:m
    for k = 1:p
        M(i, :) = M(i, :) + A(i, k) * B(k, :);
    end
end
```

Similarly, we can express each column j of $\mathbf{M}$ as a linear combination of all the columns of $\mathbf{A}$, where the coefficients come from the j-th column of $\mathbf{B}$. Again, mathematically this equates to

$$\mathbf{m}_{:j} = \sum_{k=1}^{p} b_{kj}\mathbf{a}_{:k}$$

and the MATLAB code is

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for j = 1:n
    for k = 1:p
        M(:, j) = M(:, j) + B(k, j) * A(:, k);
    end
end
```

Problem 2

Expressing each column j of $\mathbf{M}$ as the matrix-vector product of $\mathbf{A}$ with the j-th column of $\mathbf{B}$ was already given on the assignment. Expressing it in MATLAB is then

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for j = 1:n
    M(:, j) = A*B(:, j);
end
```

We can also express each row i of $\mathbf{M}$ as the vector-matrix product of the i-th row of $\mathbf{A}$ with the matrix $\mathbf{B}$. Mathematically, this is

$$\mathbf{m}_{\mathbf{i}:} = \mathbf{a}_{\mathbf{i}:}\mathbf{B}$$

and in MATLAB is

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for i = 1:m
    M(i, :) = A(i,:)*B;
end
```

The final method can be derived by reordering the triply nested matrix multiplication loop so that the loop over p (the number of columns of $\mathbf{A}$ and the number of rows of $\mathbf{B}$) appears as the outermost loop. The inner two loops are then performing a matrix multiplication between the k-th column vector of $\mathbf{A}$ and the k-th row vector of $\mathbf{B}$, producing an $[m \times n]$ matrix. These matrices are summed together to form the matrix $\mathbf{M}$. Mathematically, this can be expressed as

$$\mathbf{M} = \sum_{k=1}^{p} \mathbf{a}_{:k}\mathbf{b}_{k:}$$

The product of a column vector and a row vector is also known as the *outer product*. Be careful not to confuse this with the dot product (also known as the *inner product*), which can be thought of as the multiplication of a row vector with a column vector (note the reversed order). For the dot product, given a $[1 \times p]$ vector and a $[p \times 1]$ vector, it produces a $[1 \times 1]$ matrix – or a scalar number. For the outer product, given a $[m \times 1]$ vector and a $[1 \times n]$ vector, it produces a $[m \times n]$ matrix.

In MATLAB, this method can be expressed as

```
[m, p] = size(A);
[p, n] = size(B);
M = zeros(m, n);
for k = 1:p
    M = M + A(:,k)*B(k,:);
end
```